

# $\beta$ -decay half-lives using the ANN model: Input for the r-process



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# Origin of the Elements

- ✓ Almost all of the **H**, **He** along with some of the **Li** in nature was created in the first three minutes after the Big Bang.
- ✓ Two more light elements, **Be** and, **B** are synthesized in interstellar space by **collisions** between cosmic rays and gas nuclei (spallation).
- ✓ The rest elements up to **<sup>56</sup>Fe**, are formed by exothermic **fusion** reactions inside stellar cores.
- ✓ The heavier elements, **beyond Fe** are almost exclusively formed in **n-capture** processes (i.e. r,s, ...), avoiding Coulomb barrier and endothermic reactions (Q-values<0).

「B<sup>2</sup>FH 「1」, Cameron 「2」 - 50 years ago」

- ✓ The superheavy elements are produced via spontaneous **fission**.

**Periodic Table  
of the Elements**

<http://chemistry.about.com>  
 ©2010 Todd Helmenstine  
 About Chemistry

1A	2A															8A	
1	2															He	
H	Be																
3	4																
Li	Be																
11	12																
Na	Mg																
22.989769	24.3050																
3B	4B	5B	6B	7B	<b>8B</b>		1B	2B									
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	
39.0993	40.079	44.95912	47.867	50.9415	51.9961	54.933045	55.845	58.933195	63.546	65.38	69.723	72.64	74.92160	78.96	79.904	83.798	
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	Xe	
85.4078	87.62	88.90595	91.224	92.90638	95.96	[98]	101.07	102.90550	106.42	107.8892	112.411	114.818	116.719	121.760	127.65	131.90447	131.293
55	56	57-71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	
Cs	Ba	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
132.9056519	137.327	Lanthanides	176.49	180.94798	183.84	185.207	190.23	192.217	195.064	198.96595	200.59	204.3833	207.2	209.904045	210.9	211.9	212.9
87	88	89-103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	
Fr	Ra	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Uuq	Uup	Uuh	Uus	Uuo	
[223]	[226]	Actinides	[267]	[268]	[271]	[272]	[270]	[276]	[281]	[286]	[284]	[288]	[286]	[288]	[288]	[294]	[294]
Lanthanides																	
57	58	59	60	61	62	63	64	65	66	67	68	69	70	71			
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu			
138.90547	140.116	140.90765	144.242	[145]	150.38	151.954	157.25	158.92535	162.500	164.93032	167.259	168.93421	173.054	174.9968			
89	90	91	92	93	94	95	96	97	98	99	100	101	102	103			
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr			
[227]	232.03806	231.03589	238.02891	[237]	[244]	[243]	[247]	[247]	[251]	[252]	[257]	[258]	[259]	[262]			

「1」 E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Rev. Mod. Phys. 29, 547 (1957)

「2」 A. G. W. Cameron, Chalk River Lab. Rep. CRL-41, A.E.C.L. No. 454, Ontario (1957)

# The R-Process Nucleosynthesis

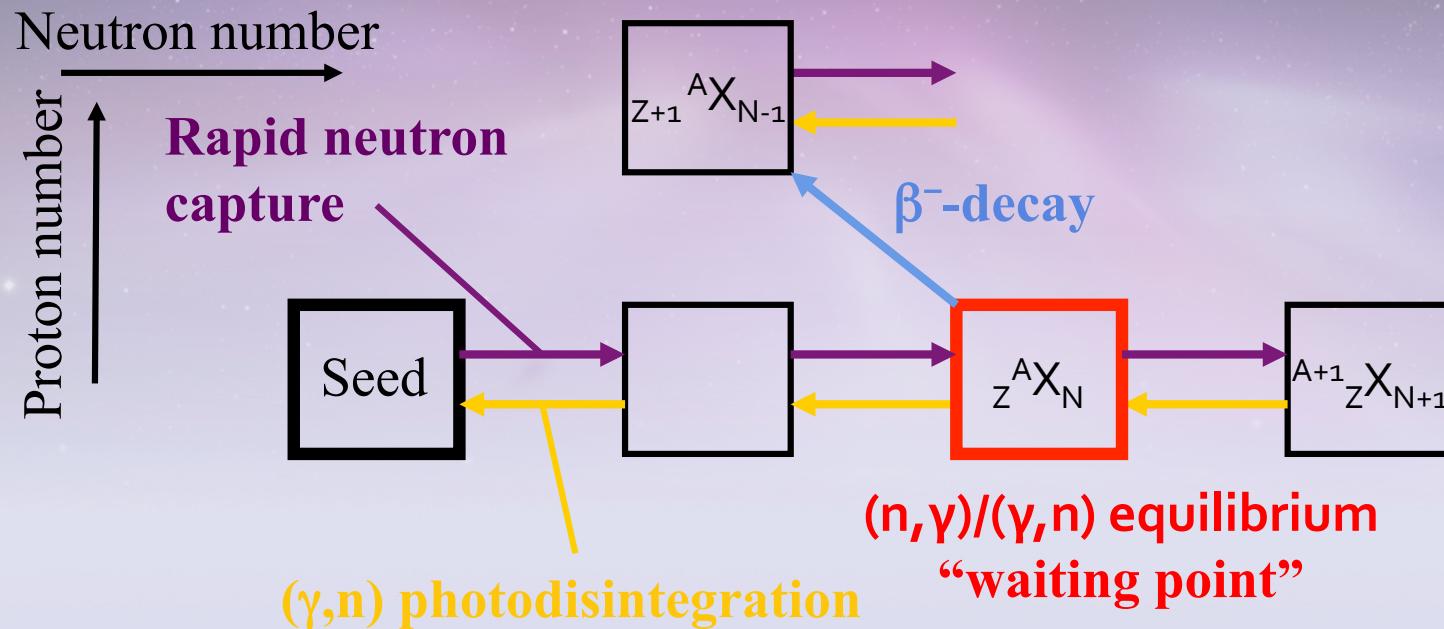
R → Rapid neutron-capture

$T \sim 1\text{-}2 \text{ GK}$ ,  $n_n \sim 10^{20}\text{-}10^{26} / \text{cm}^3$

Density:  $300 \text{ g/cm}^3$  ( $\sim 60\%$  neutrons !)

n-capture timescale ( $\tau_n$ ):  $\sim 0.2 \mu\text{s}$

$$\tau_n \ll \tau_{\beta^-}$$

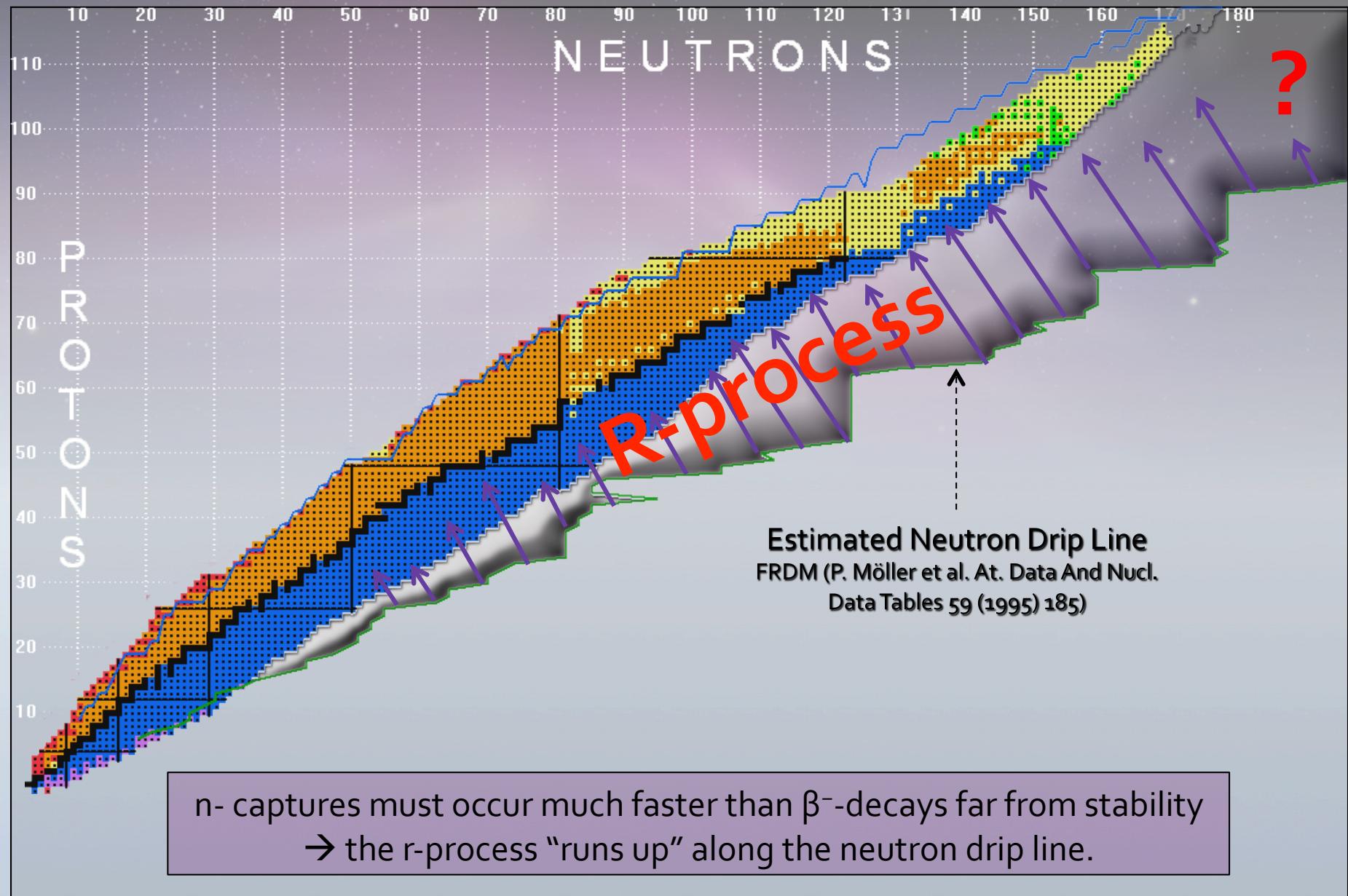


The rapid n-capture process (r-process) is today understood to be responsible for the synthesis of about half of all the nuclei abundances present in Solar System matter in the mass region from approximately Zinc through the Actinides (e.g., Thorium, Uranium, and Plutonium), as well as the bulk of the heavy elements in the mass region  $A \geq 60$  in our Galactic and Cosmos matter.

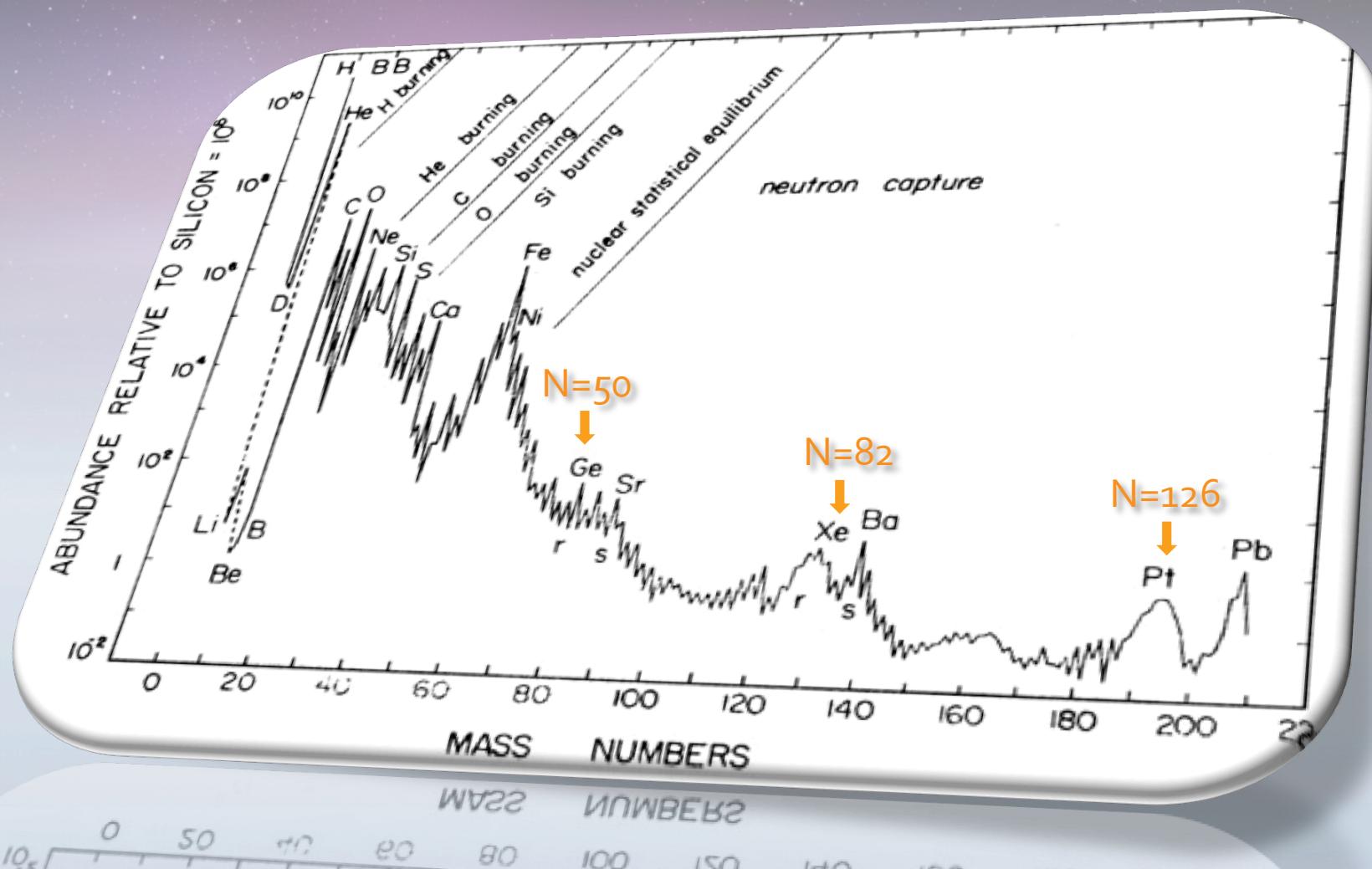
R-process still remains one of the most challenging open questions in modern nuclear astrophysics, since both the astrophysical scenarios where this process occurs and the needed nuclear physics input have not yet been unambiguously identified.

# R-Process Path

chart of nuclides



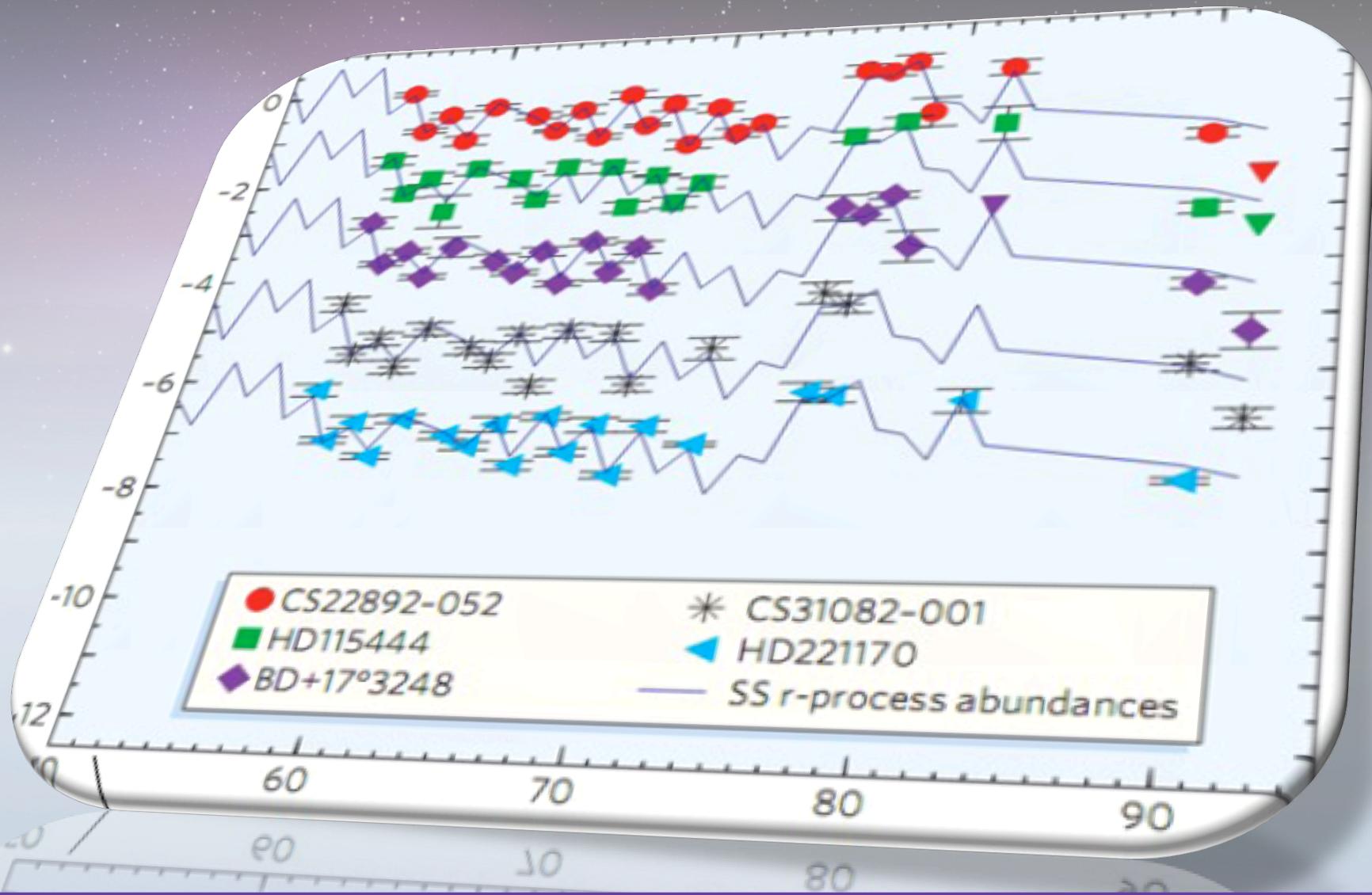
# Relative Solar System Abundances



The solar elemental abundance distribution beyond Fe, shows peaks near  $A = 80, 130$ , and  $195$ , corresponding to progenitors with closed neutron shells  $N = 50, 82$ , and  $126$ .

# Old Metal Poor Galactic Halo Stars

The r-process operating over the history of the Galaxy



→ R-process is unique in nature!

# Understanding of R-Nucleosynthesis

r-abundances reproduction

N U C L E A R A S T R O P H Y S I C S .

## Astrophysical Conditions

temperature ( $T_9$ )

matter density

neutron density ( $n_n$ )

...

radiation entropy ( $S_{rad}$ )

r-process duration ( $\tau_r$ )

## Nuclear Physics Input

nuclear-masses ( $Q_\beta, S_n$ )

$\beta$ -decay properties ( $T_{1/2}, P_n$ )

n-capture rates

fission properties

neutrino interactions

nuclear-structure-properties ( $J^\pi \dots$ )

for 1000's of isotopes FAR-OFF  $\beta$ -stability

# Astrophysical Site (s)

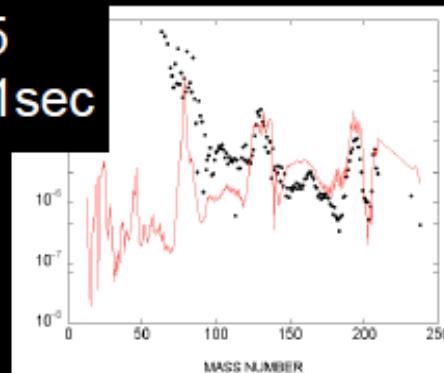
Dominant Candidates

for the “main” r-process

## Neutrino-driven wind in SNe II

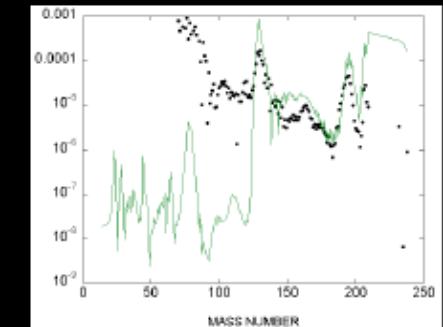
SN1987A

$S \sim 400$   
 $Y_e \sim 0.45$   
 $\tau_{dyn} \sim 0.1 \text{ sec}$



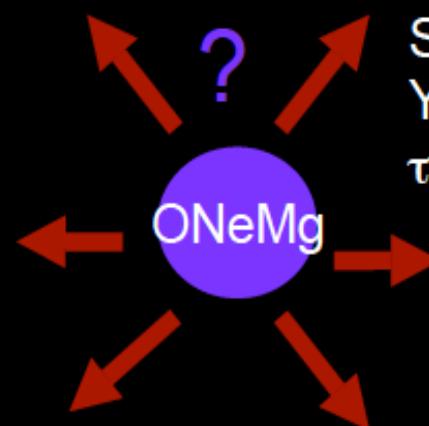
## Neutron star mergers

Simulation of NS mergers  
(from Hayden planetarium)

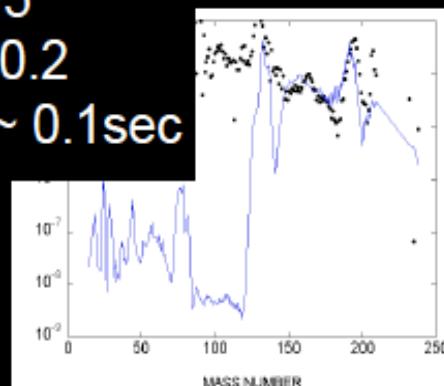


$S < 4$   
 $Y_e < 0.2$

## Prompt explosion of low mass supernovae



$S \sim 15$   
 $Y_e \sim 0.2$   
 $\tau_{dyn} \sim 0.1 \text{ sec}$



The most widely believed candidate site for the r-process are core-collapse supernovae (spectral Type Ib, Ic and II), which provide the necessary physical conditions for the r-process. However, the abundance of r-process nuclei requires that either only a small fraction of supernovae eject r-process nuclei to the interstellar medium, or that each supernova ejects only a very small amount of r-process material. A recently proposed alternative solution is that neutron star mergers (a binary star system of two neutron stars that collide) may also play a role in the production of r-process nuclei, but this has yet to be observationally confirmed.

If nuclear physics uncertainties are reduced, we could identify astrophysical site via observations.

# Understanding of R-Nucleosynthesis

r-abundances reproduction

N U C L E A R A S T R O P H Y S I C S .

## Astrophysical Conditions

temperature ( $T_9$ )

matter density

neutron density ( $n_n$ )

...

radiation entropy ( $S_{rad}$ )

r-process duration ( $\tau_r$ )

## Nuclear Physics Input

nuclear-masses ( $Q_\beta, S_n$ )

$\beta$ -decay properties ( $T_{1/2}, P_n$ )

n-capture rates

fission properties

neutrino interactions

nuclear-structure-properties ( $J^\pi \dots$ )

for 1000's of isotopes FAR-OFF  $\beta$ -stability

# Nuclear Physics Input

$T_{1/2}$  of very neutron-rich nuclei

## Nuclear masses

- $S_n$ -values  $\Rightarrow$  r-process paths (i.e.  $S_n \sim 3$  MeV)

## $\beta$ -decay properties

$\beta^-$  Half-lives  $\Rightarrow$  duration – speed – time scale  
 $\Rightarrow$  r-process clock

$T_{1/2}$  (g.s., is.)  $\Rightarrow$  r-process progenitor abundances ( $N_{r,\text{prog}}$ )

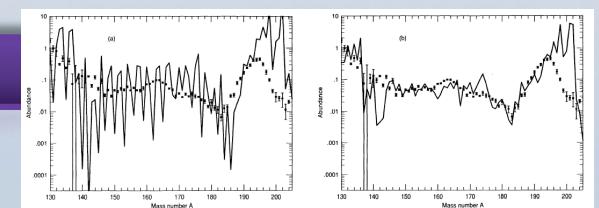
$T_{1/2}$  of waiting points  $\Rightarrow$  pre freeze-out isobaric abundances and regulate the speed of the process towards heavier elements.

$T_{1/2}$  of heavy neutron-rich radionuclides  $\Rightarrow$  time scale for the matter flow from the r-process seeds to even heavier nuclei.

Sum of  $T_{1/2}$   $\Rightarrow$  total r-process duration ( $\tau_r$ )

## $\beta$ -delayed n-emission branching probabilities

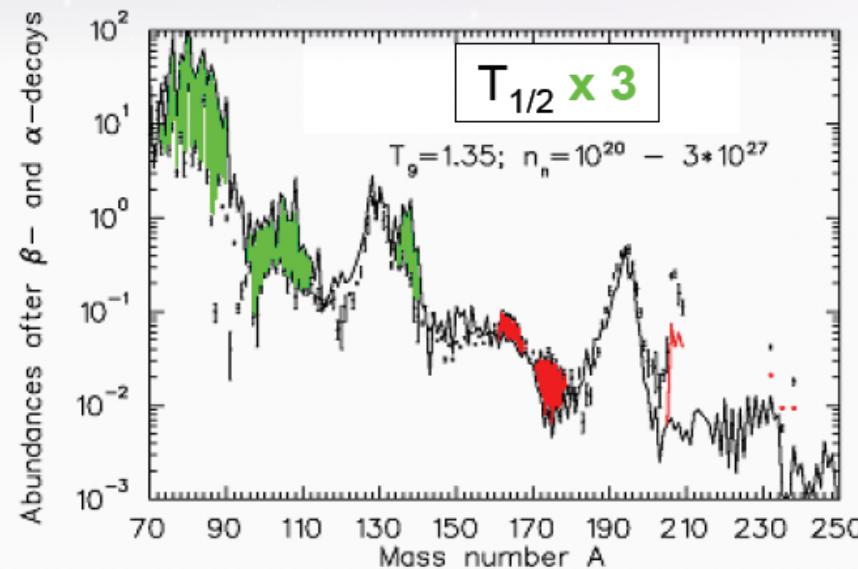
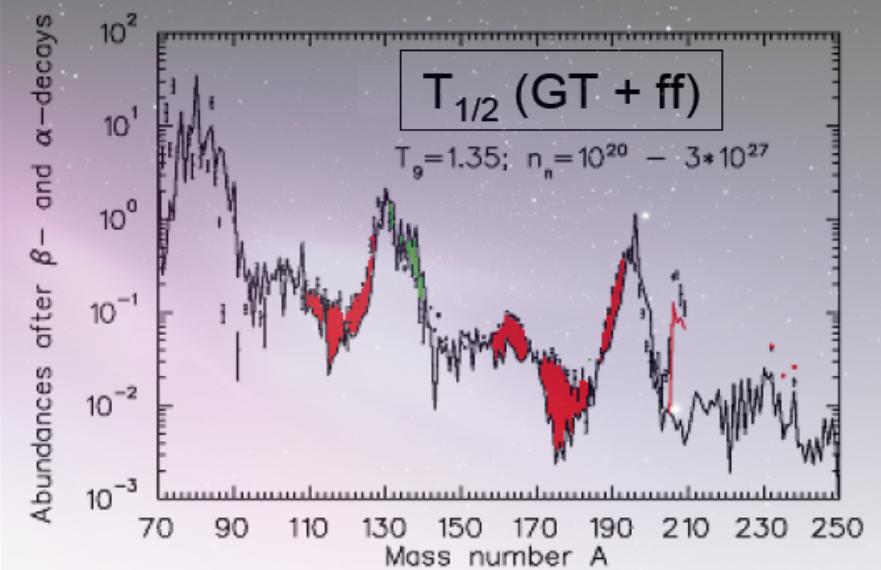
$P_n \Rightarrow$  smoothing  $N_{r,\text{prog}} \rightarrow N_{r,\text{final}} (N_{r,\odot})$



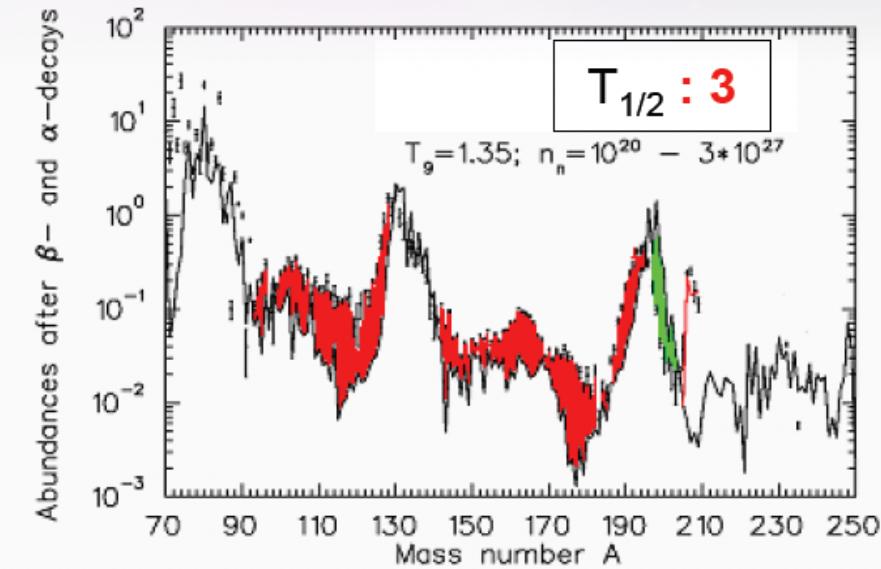
# Effects of $T_{1/2}$ on matter flow

Mass model: **ETFSI-Q**  
-all astro parameters kept constant

r-process model:  
**“waiting-point approximation”**

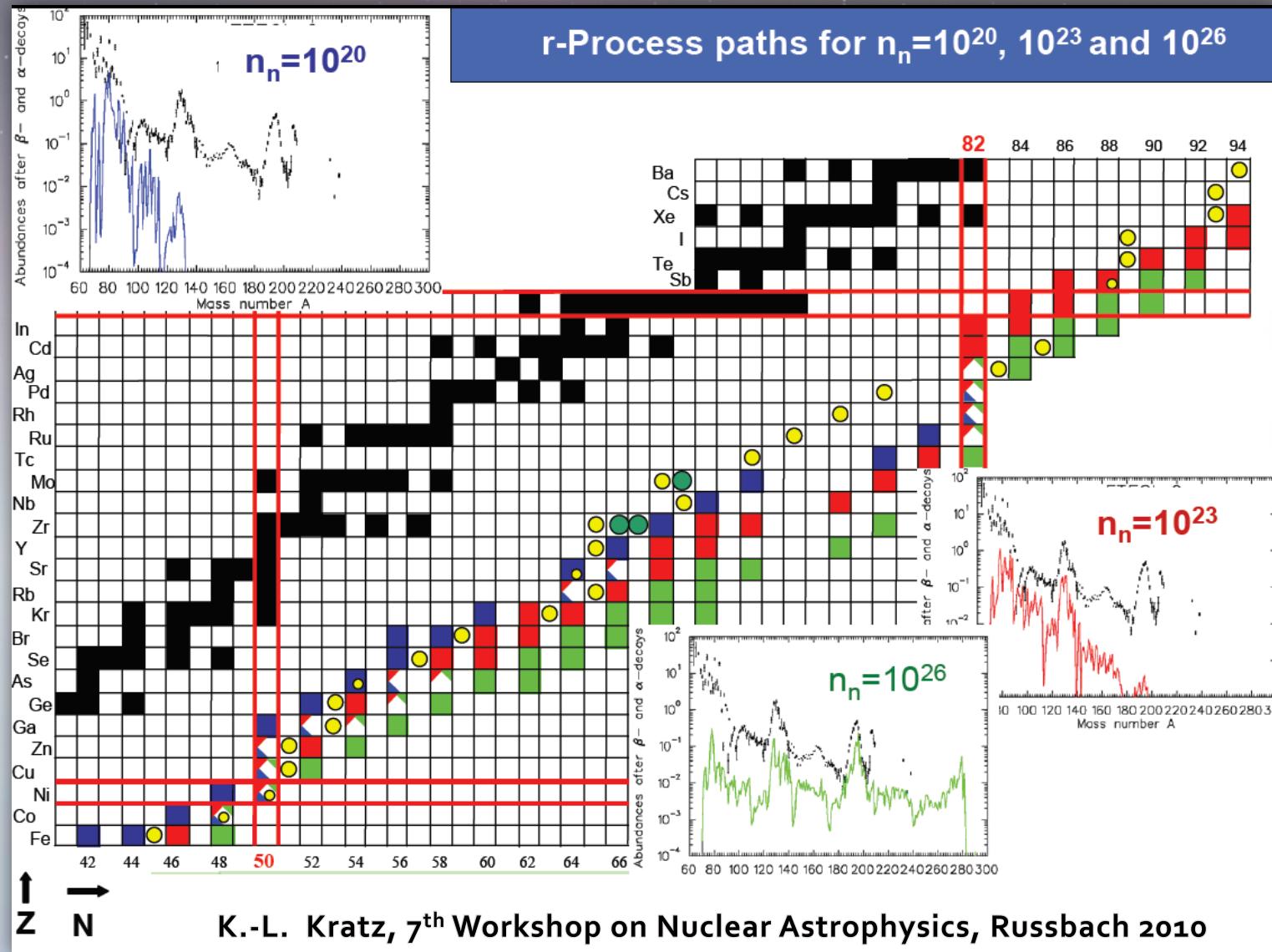


r-matter flow too **slow**



r-matter flow too **fast**

# Today's Exp. Info - R-process paths



Today, altogether  
60 r-process nuclei  
are only known!

- Heaviest isotopes with measured  $T_{1/2}$
- new (MSU, 2009)

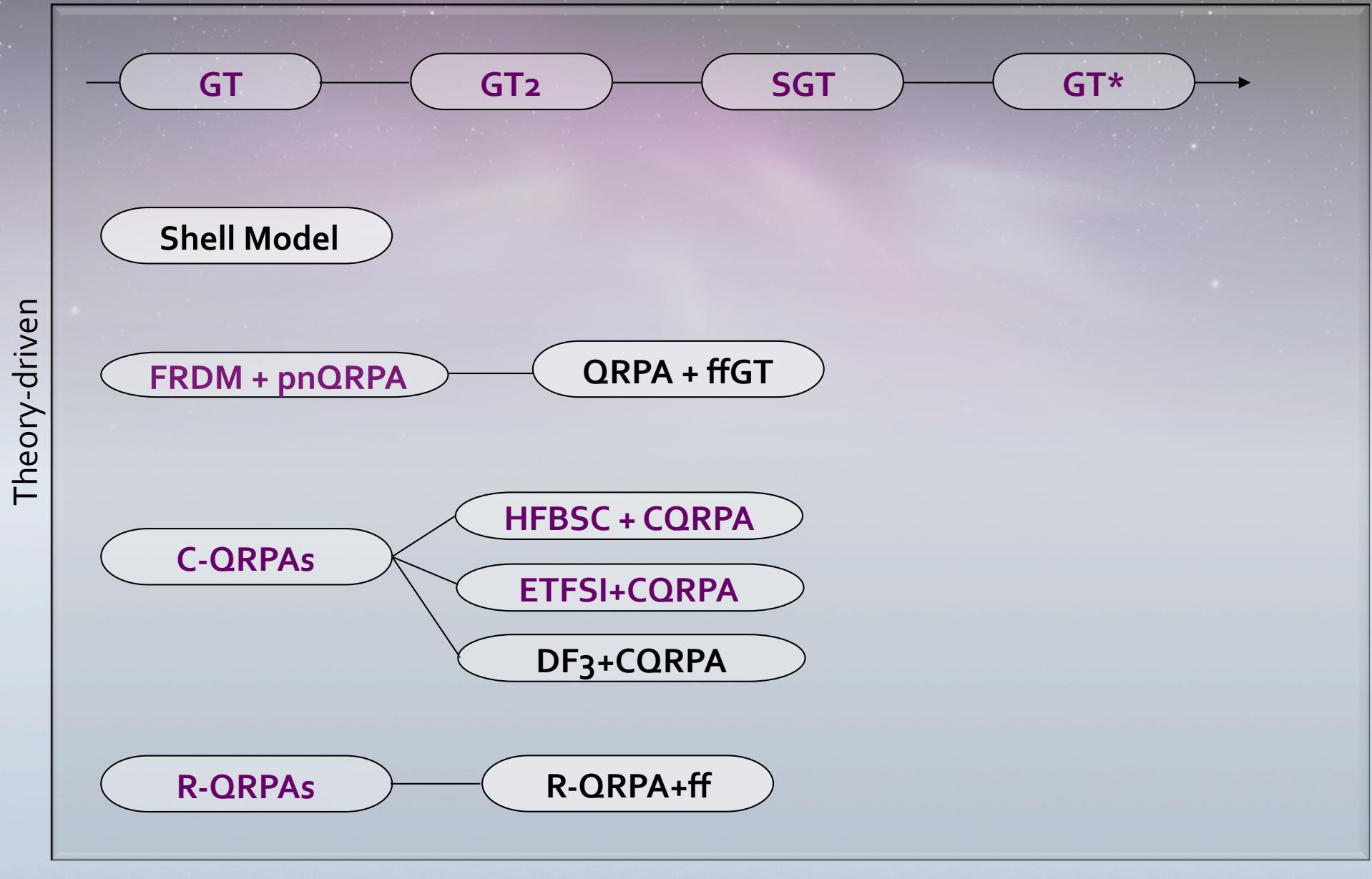
"Waiting point" isotopes at:

- $n_n = 10^{20}$
- $n_n = 10^{23}$
- $n_n = 10^{26}$

freeze-out networks.

The r-process should be a dynamical process with continuously altered conditions and paths.

# $\beta^-$ -decay Half-lives Approaches



# Statistical Modeling

Learning Machines

Data- driven

Artificial Neural Networks (ANNs)

Support Vector Machines (SVMs)

Ref: V. N. Vapnik, Statistical Learning Theory, Wiley-Interscience , 1998.

# Statistical Modeling Using ANNs

data-driven, stand-alone statistical approaches

## Fundamental Question

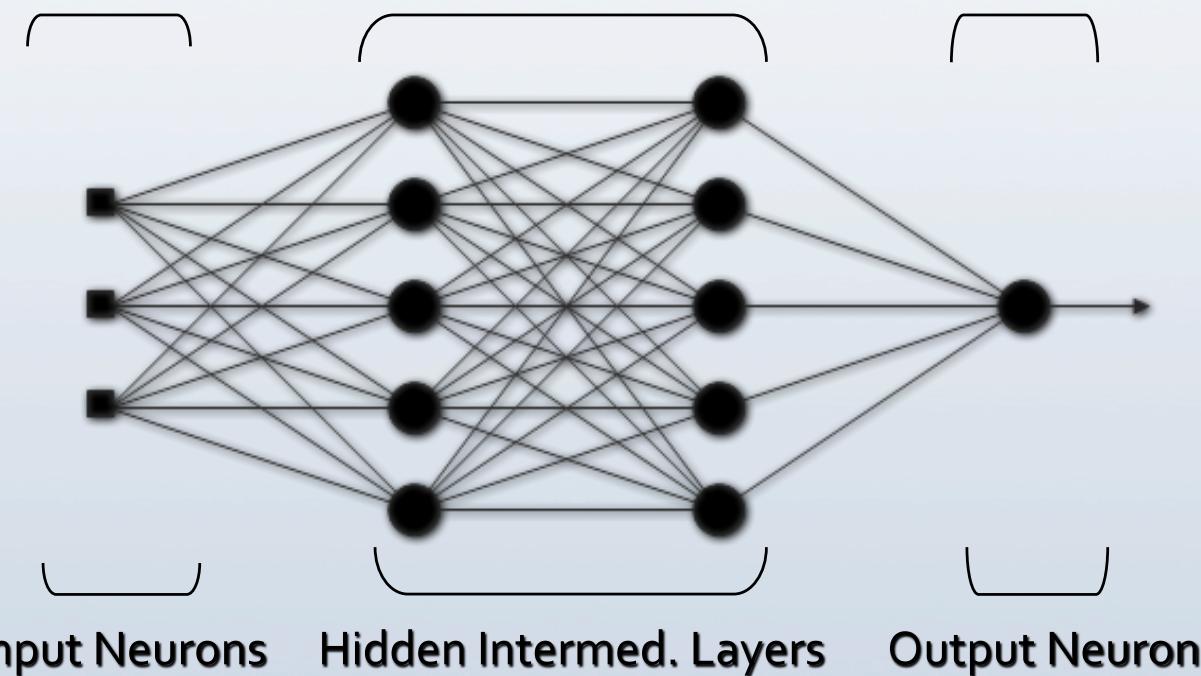
To what extend can mature learning machines (i.e. ANNs) decode the underlying  $\beta$ -decay half-lives systematics by **only** utilizing the hidden information over the **minimal input** of the **Z** and **N** nucleonic numbers?

# Artificial Neural Networks

## Learning Machines

classification – function approximation

### Fully-connected Feed-forward Neural Network



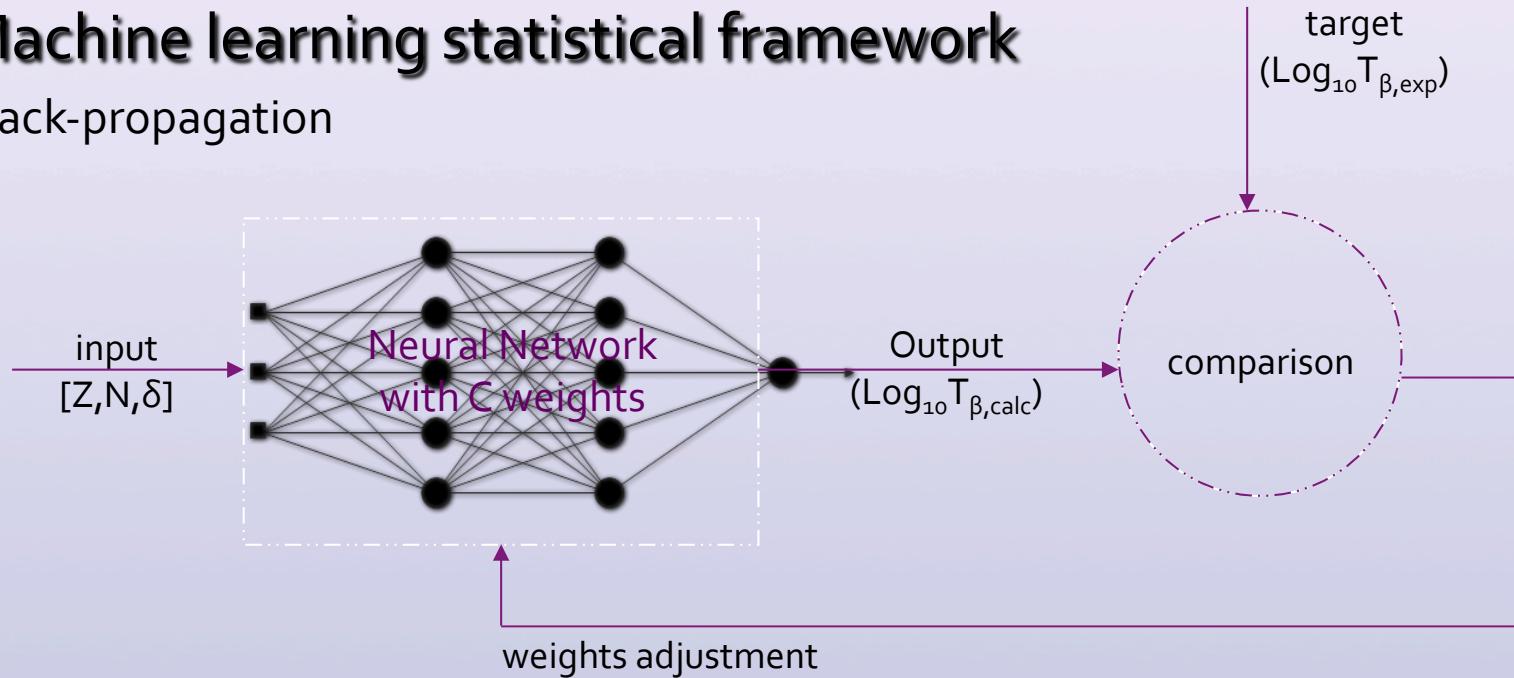
# Learning Procedure

Objective: minimization of cost function

$$E_D = \sum_{i=1}^N \left( \log_{10} \left( \frac{T_i}{T_{\beta, \text{exp}}} \right) - \log_{10} \left( \frac{T_i}{T_{\beta, \text{calc}}} \right) \right)^2$$

Machine learning statistical framework

Back-propagation



Levenberg-Marquardt BP update rule

$$\vec{w}_{k+1} = \vec{w}_k - [\vec{J}_k^T \vec{J}_k + \epsilon]^{-1} \vec{J}_k^T \vec{e}_k$$

# The ANN Model

## General ANN Model Features

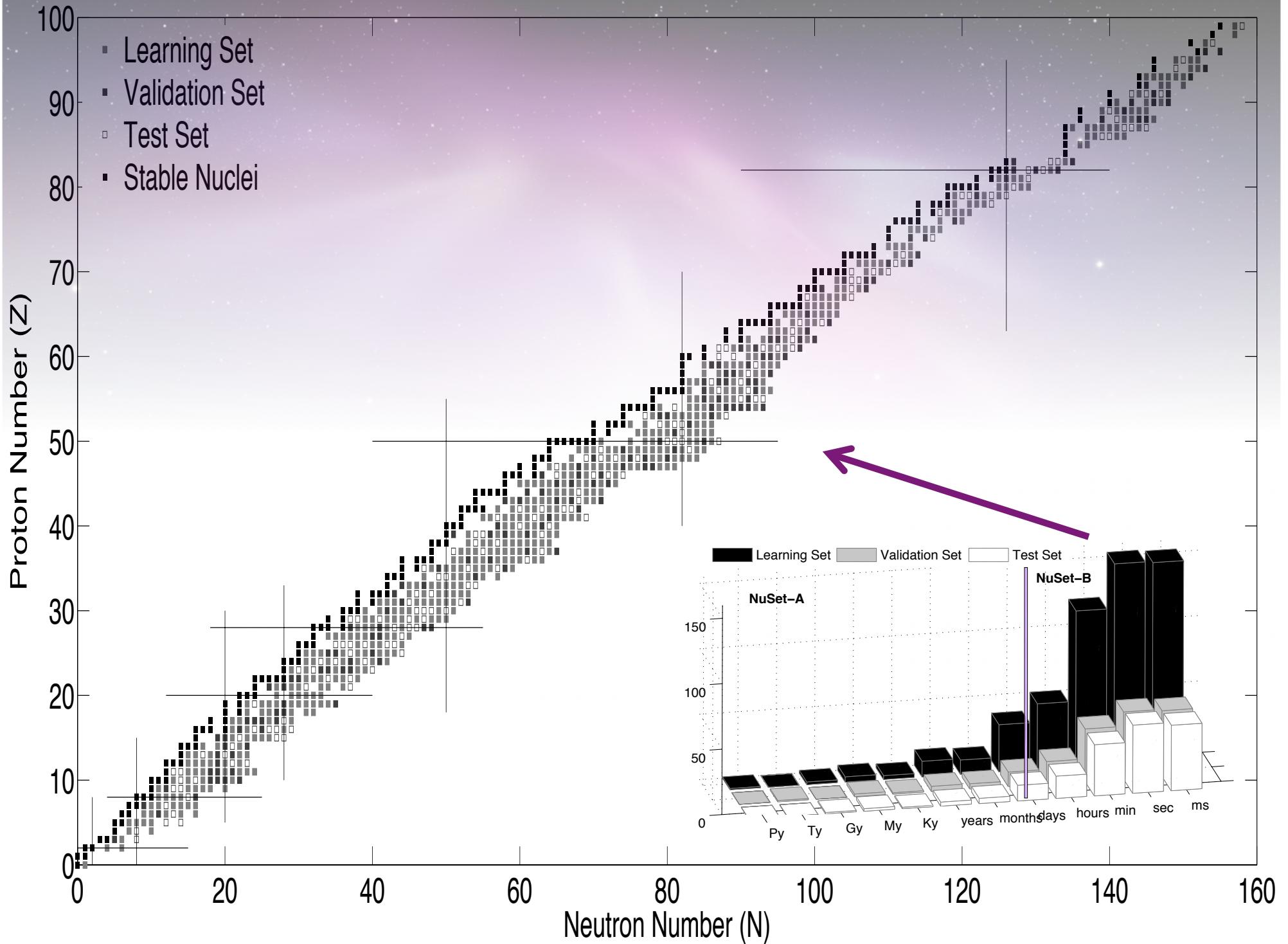
Network	Architecture		Activation Functions
Feed-forward Fully-connected	<b>Size:</b>	3-5-5-5-5-1	tanh-tanh-tanh- tanh-satlins
	<b>Weights:</b>	116	
Mode	Training Algorithm		Initialization Method
Batch	Bayesian Regularization		Nguyen-Widrow
Input Scaling [-1, 1]		Output Scaling [-1, 1]	Max Fail
Z	[0, 230]	$\log_{10} T_{1/2}$ (ms)	300 epochs
N	[0, 230]		
Parity	[-1, 1]		

<b>Database</b>	<b>Half-life Range</b>	<b>Decay Mode</b>	<b>NuSet-B</b>	
<b>NuBase2003 *</b>	$0.15 \times 10^{-2} \text{ s} \rightarrow {}^{35}\text{Na}$ $2.43 \times 10^{23} \text{ s} \rightarrow {}^{113}\text{Cd}$	100% $\beta^-$	<b>Cutoff</b>	$10^6 \text{ s}$

\* A.H. Wapstra, G. Audi et al., Nucl Phys. A729 (2003) 337

To avoid overfitting effects		
Early Stopping		
<b>Training Set</b>	<b>Validation Set</b>	<b>Test Set</b>

<b>NuSet-B. Uniform Nuclides Distribution over Half-lives</b>			
<b>All Nuclides</b>	<b>Training Set (60%)</b>	<b>Validation Set (20%)</b>	<b>Test Set (20%)</b>
843	503	167	168



# ANN Performance

Set	RMSE	fact	$T_{1/2}$ Exp (s)	Overall Mode			NBSC+pnQRPA			
				m(%)	$\langle x \rangle_K$	$\sigma_K$	m(%)	$\langle x \rangle_K$	$\sigma_K$	
Learning	0.53	<10	<10 <sup>6</sup>	90.5	2.46	1.72	76.7	3.00	-	
Validation	0.60		<60	96.5	2.21	1.52	87.2	2.81	-	
Test	0.65		<1	97.6	2.10	1.39	95.7	2.64	-	
Overall	0.57		<10 <sup>6</sup>	53.5	1.41	0.27	33.8	1.43	-	
			<60	60.6	1.41	0.27	42.0	1.41	-	
			<1	61.9	1.41	0.26	50.7	1.43	-	

$RMSE = \sqrt{\frac{1}{N} \sum_i (\log_{10} \beta_{calc} - \log_{10} \beta_{exp})^2}$

$$\bar{x}_K = \frac{1}{N} \sum_i x_i, \quad x_i = \begin{cases} T_{\beta,exp}/T_{\beta,calc} & \text{if } T_{\beta,exp} \geq T_{\beta,calc} \\ T_{\beta,calc}/T_{\beta,exp} & \text{if } T_{\beta,exp} < T_{\beta,calc} \end{cases} \quad \sigma_K = \left[ \frac{1}{N} \sum_i (x_i - \bar{x}_K)^2 \right]^{1/2}$$

# Results

Recently measured  $T_\beta$  for very neutron-rich nuclides.

Nucleus	Experiment	$T_\beta$ (ms)	
		ANN Model	<i>pnQRPA+ffGT</i>
$^{36}\text{Mg}$	$3.9 \pm 1.3$	14.5	15.9
$^{37}\text{Al}$	$10.7 \pm 1.3$	15	9.9
$^{38}\text{Al}$	$7.6 \pm 0.6$	7.7	4.7
$^{39}\text{Al}$	$7.6 \pm 1.6$	7.2	4.3
$^{39}\text{Si}$	$47.5 \pm 2.0$	27.7	101.5
$^{40}\text{Si}$	$33.0 \pm 1.0$	48.5	30.6
$^{42}\text{Si}$	$12.5 \pm 3.5$	15.5	43.4
$^{44}\text{P}$	$18.5 \pm 2.5$	12.7	17.2
$^{46}\text{S}$	$50 \pm 8$	39.5	30.8
$^{47}\text{Cl}$	$101 \pm 6$	77.9	51.5
$^{48}\text{Ar}$	$475 \pm 40$	447.6	181.9
$^{49}\text{Ar}$	$170 \pm 50$	130.2	54.9
$^{64}\text{V}$	$19 \pm 8$	23.4	7.6
$^{73}\text{Co}$	$41 \pm 4$	103.6	30.7
$^{115}\text{Tc}$	$73^{+32}_{-22}$	84.2	70.7
$^{118}\text{Ru}$	$123^{+48}_{-35}$	69.1	211.8
$^{120}\text{Rh}$	$136^{+14}_{-13}$	196.2	82.7
$^{121}\text{Rh}$	$151^{+67}_{-58}$	90.7	62.3
$^{122}\text{Pd}$	$175 \pm 16$	227.2	951.2
$^{124}\text{Pd}$	$38^{+38}_{-19}$	124.2	288.7
$^{163}\text{Eu}$	$7.8 \pm 5$ (s)	7.8 (s)	17.2 (s)
$^{164}\text{Eu}$	$4.2 \pm 2$ (s)	3.3 (s)	8.6 (s)
$^{165}\text{Eu}$	$2.3 \pm 2$ (s)	3.1 (s)	5.7 (s)
$^{199}\text{Ir}$	$6^{+5}_{-4}$ (s)	73 (s)	370.6 (s)
$\sigma_{\text{rms}}$		0.31	0.53

# Relevant to R-process Results

Some r-process waiting-point nuclei at N = 50 and N = 82 regions.

Nuc.	Exp. Data	ANN Model	$T_{\beta^-}$ (ms)	$pnQRPA+ffGT$	$pn-RQRPA+ff$	DF3+CQRPA
$^{78}\text{Ni}_{50}$	$110^{+100}_{-60}^{\text{a}}$	57	224		150	108
$^{79}\text{Cu}_{50}$	$188 \pm 25$	115	157		-	257
$^{80}\text{Zn}_{50}$	$545 \pm 16$	371	$1.26(s)$		970	839
$^{128}\text{Pd}_{82}$	-	81	74		63	43
$^{129}\text{Ag}_{82}$	$44 \pm 7$	77	32		-	56
$^{130}\text{Cd}_{82}$	$162 \pm 7$	158	502		299	147
$^{131}\text{In}_{82}$	$280 \pm 30$	307	139		-	201
$^{132}\text{Sn}_{82}$	$39.7 \pm 0.5(s)$	$30(s)$	$23.8(s)$		472.5(s)	29.8(s)

<sup>a</sup> Recent first time experimentally deduced half-life value at the CCF of the NSCL at MSU by Hosmer et al.

Recently measured\*  $T_{\beta^-}$  of eight heavier abuse to ff neutron-rich isotopes close to the neutron shell N = 126 around A = 195.

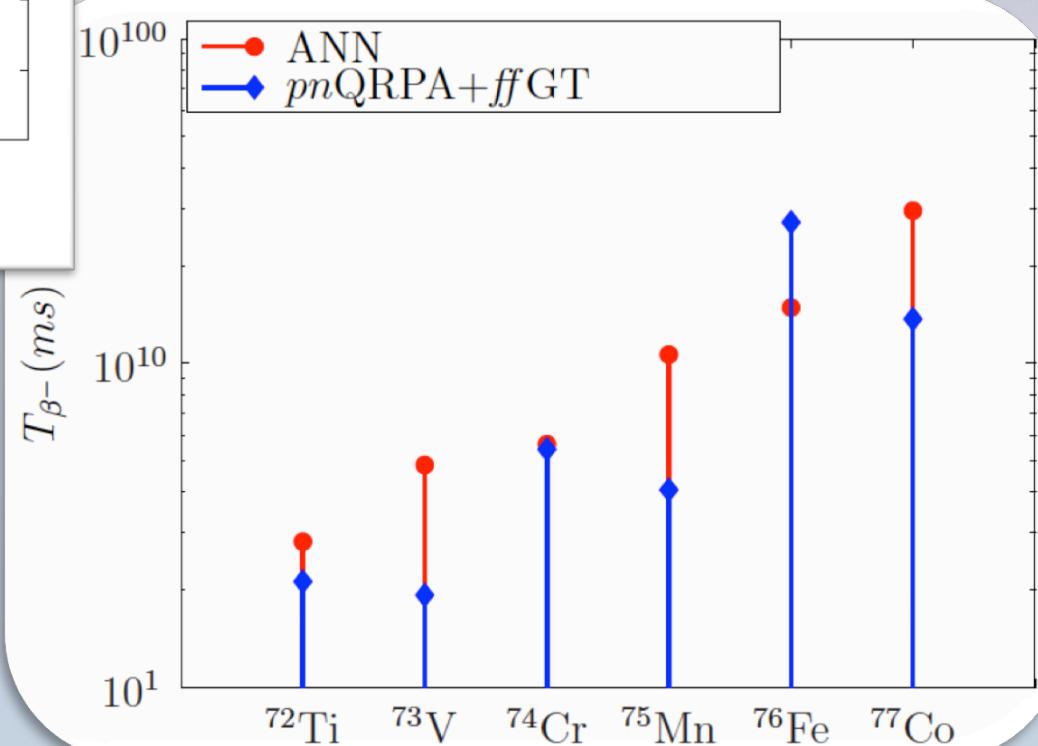
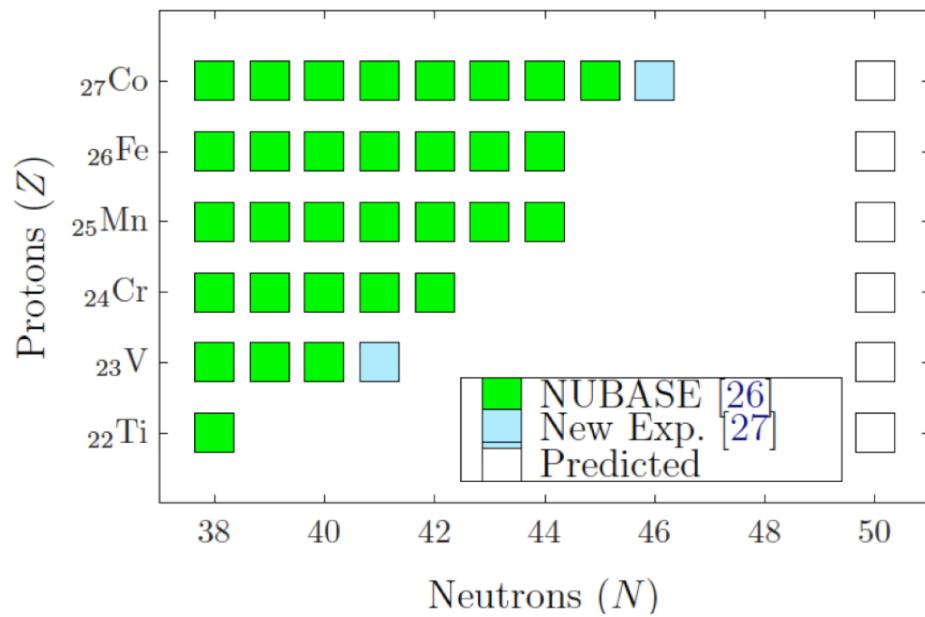
Nucleus	Exp. Data	ANN Model	$pnQRPA+ffGT$	DF3+CQRPA	Gr. Th.
$^{194}\text{Re}$	$1^{+0.5}_{-0.5}$	20.8	70.8	2.1	16.1
$^{195}\text{Re}$	$6^{+1}_{-1}$	23.9	3.3	8.5	10.3
$^{196}\text{Re}$	$3^{+1}_{-2}$	8.8	3.6	1.4	5.1
$^{199}\text{Os}$	$5^{+4}_{-2}$	13.6	106.8	6.6	17.2
$^{200}\text{Os}$	$6^{+4}_{-3}$	21.7	187.1	6.9	16
$^{198}\text{Ir}$	$8^{+2}_{-2}$	57.6	377.1	19.1	-
$^{199}\text{Ir}$	$6^{+5}_{-4}$	73	370.6	46.7	96.6
$^{202}\text{Ir}$	$11^{+3}_{-3}$	8.6	68.4	9.8	8.5
$\sigma_{\text{rms}}$	0.77	1.33	0.39	-	

\*T. Kurtukian-Nieto et al.

# N=50 Closed Shell

A = 70 – 80 mass region

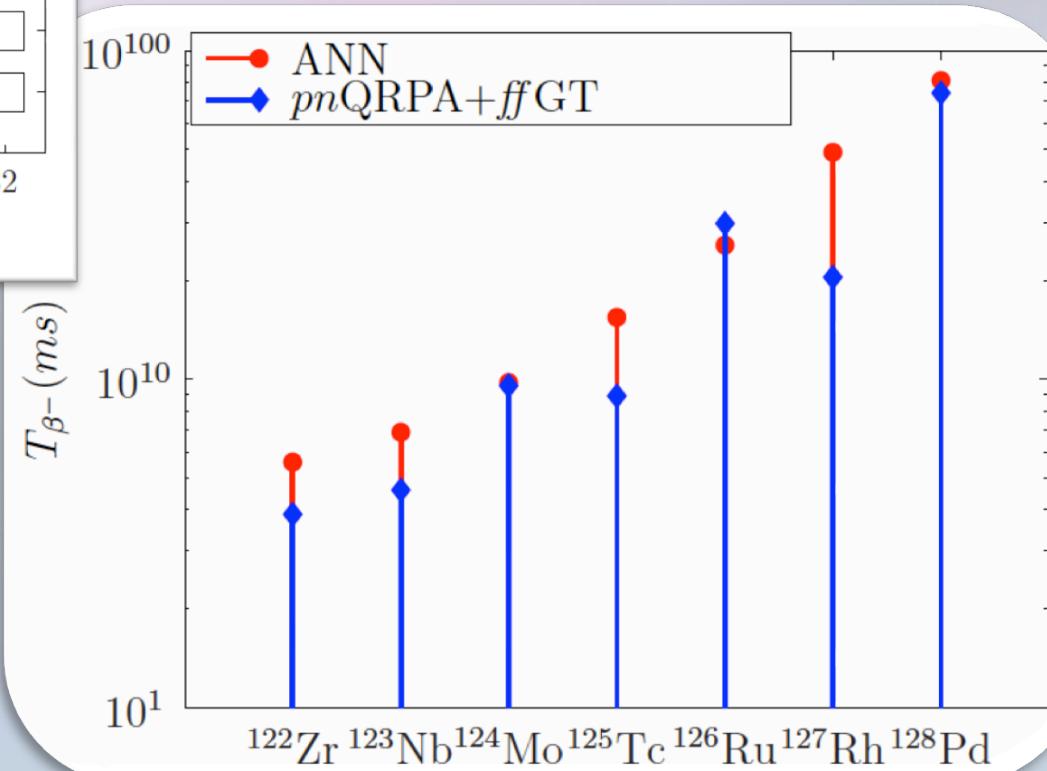
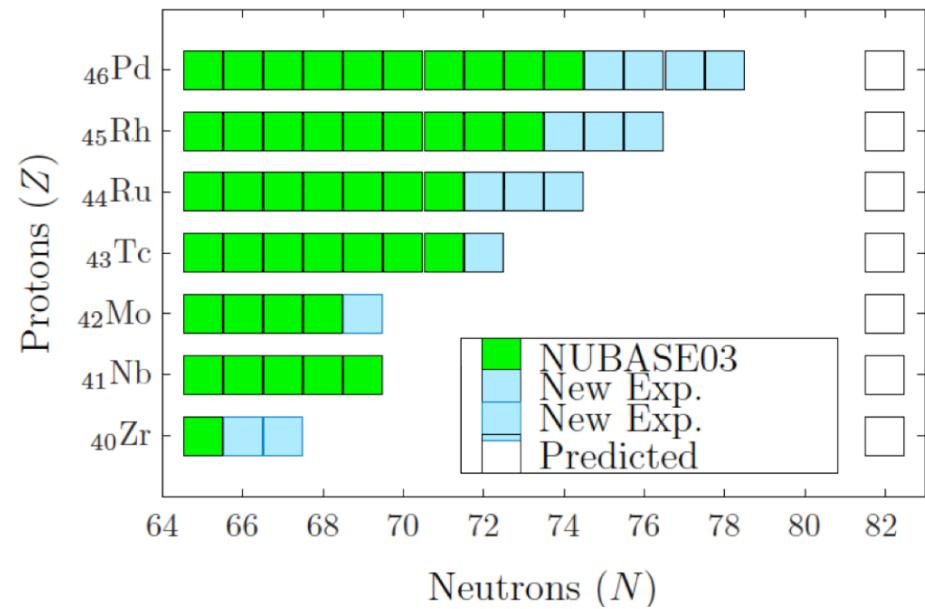
Chart of nuclides up to  $N = 50$



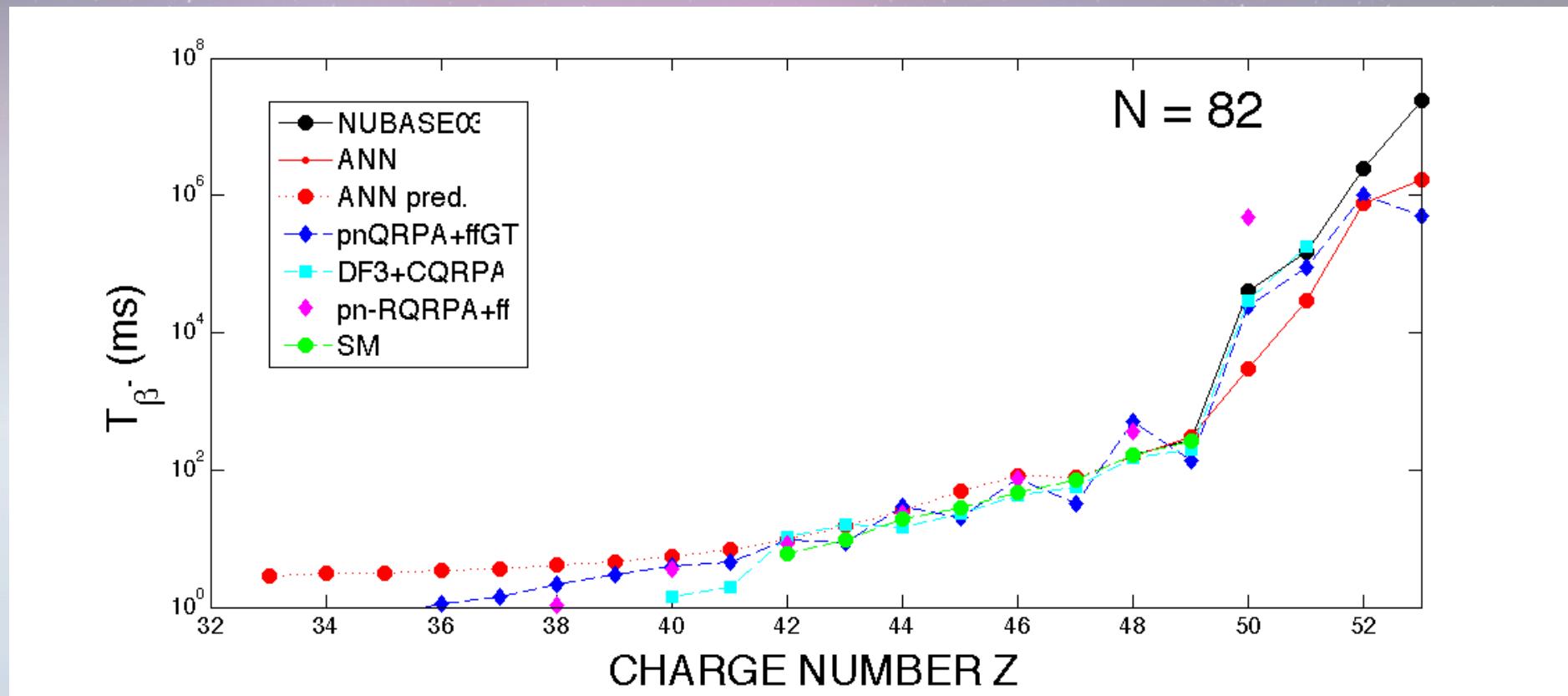
# N=82 Closed Shell

A = 120 – 130 mass region

Chart of nuclides up to  $N = 82$



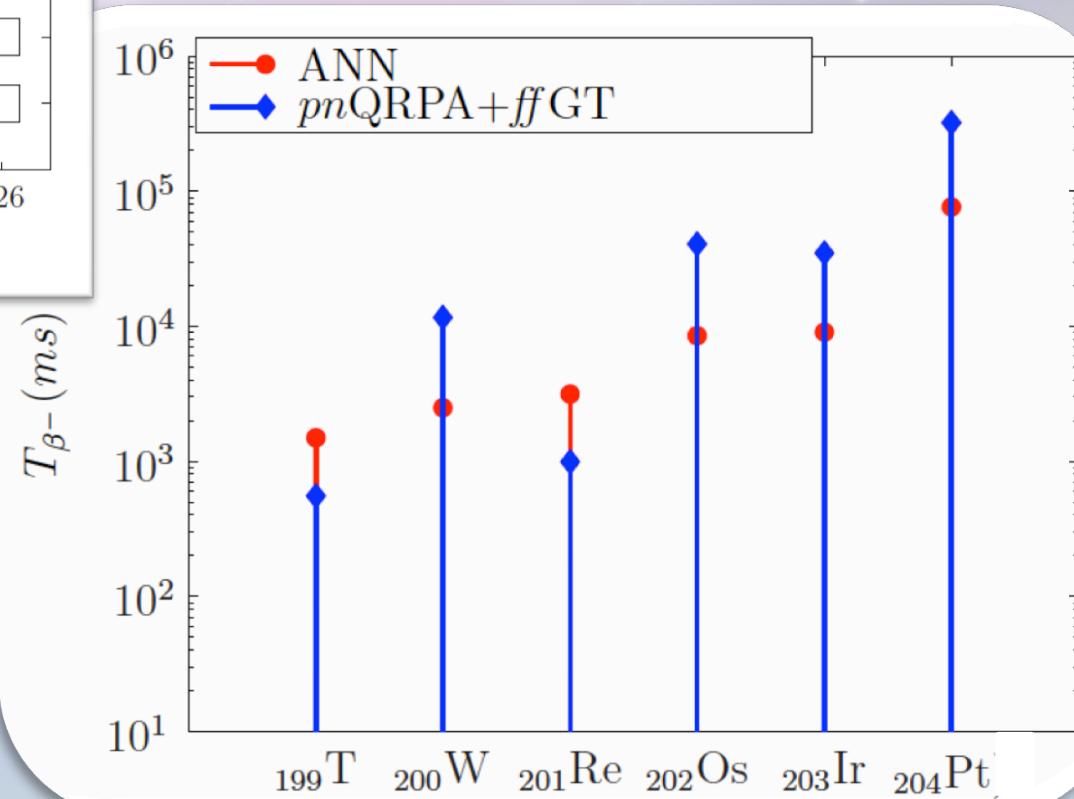
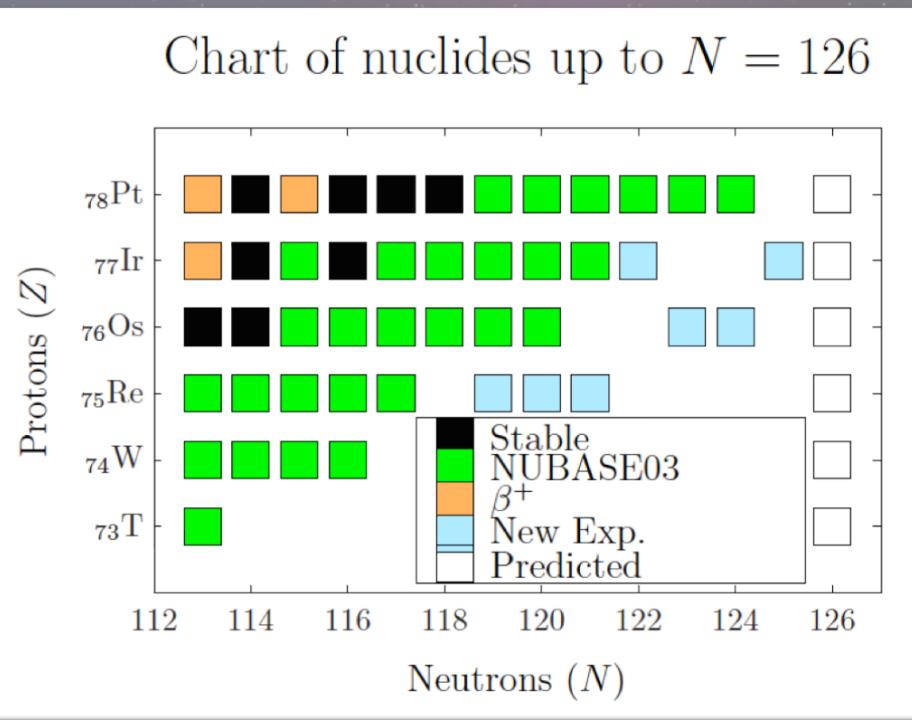
# N=82 Isotonic Chain



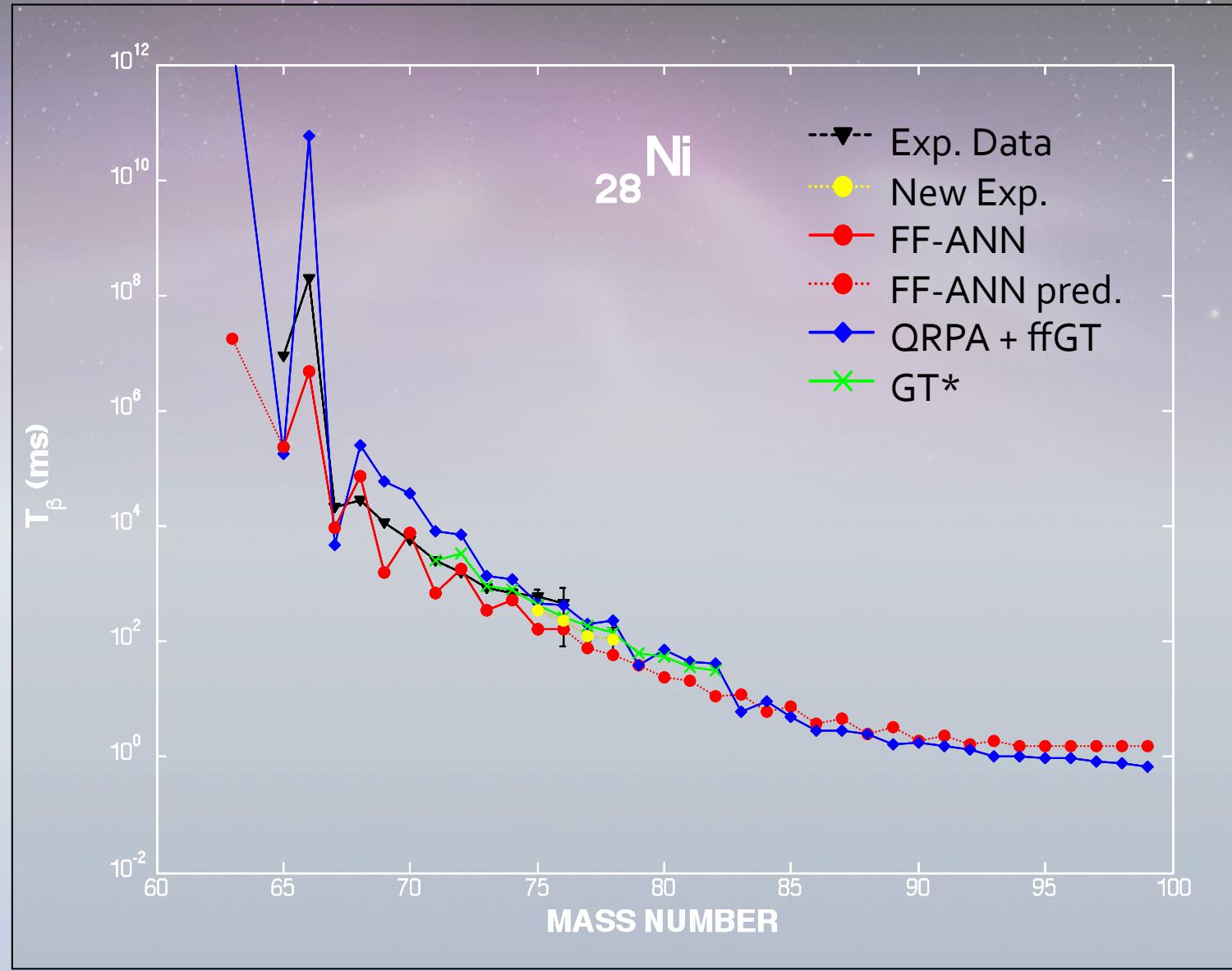
# N=126 Closed Shell

A = 195 – 205 mass region

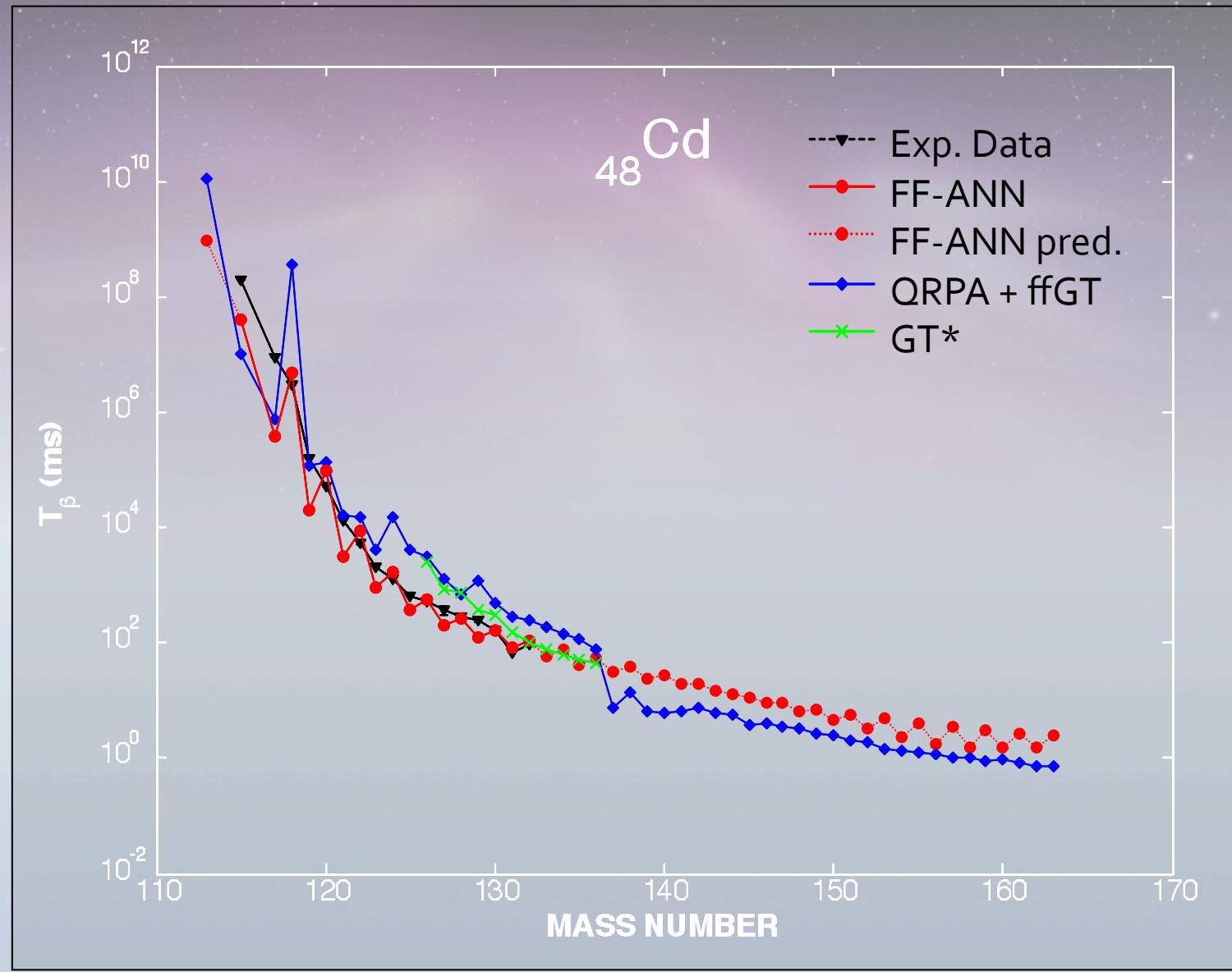
Chart of nuclides up to  $N = 126$



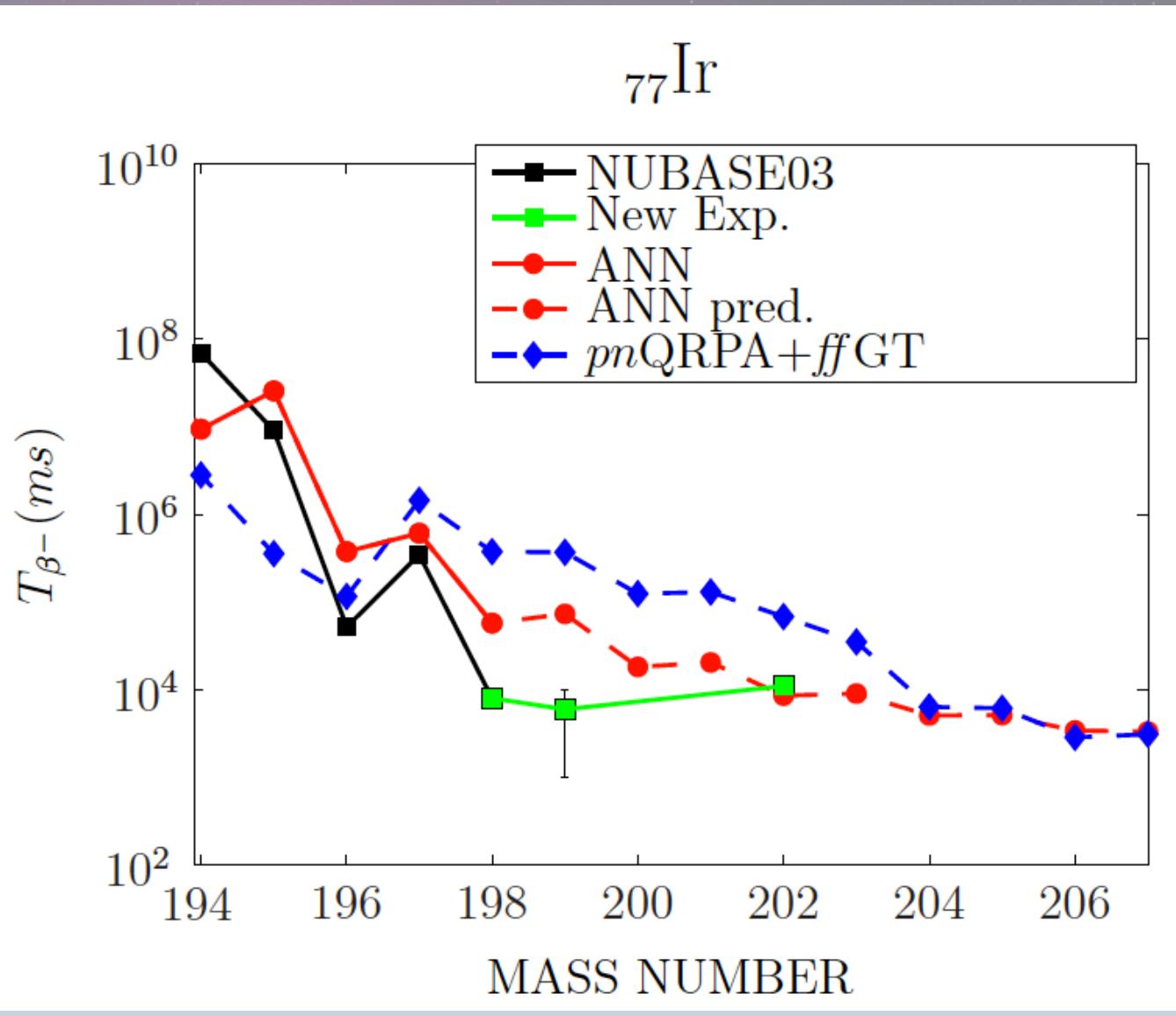
# Isotopic Chain of $^{28}\text{Ni}$



# Isotopic Chain of $^{48}\text{Cd}$

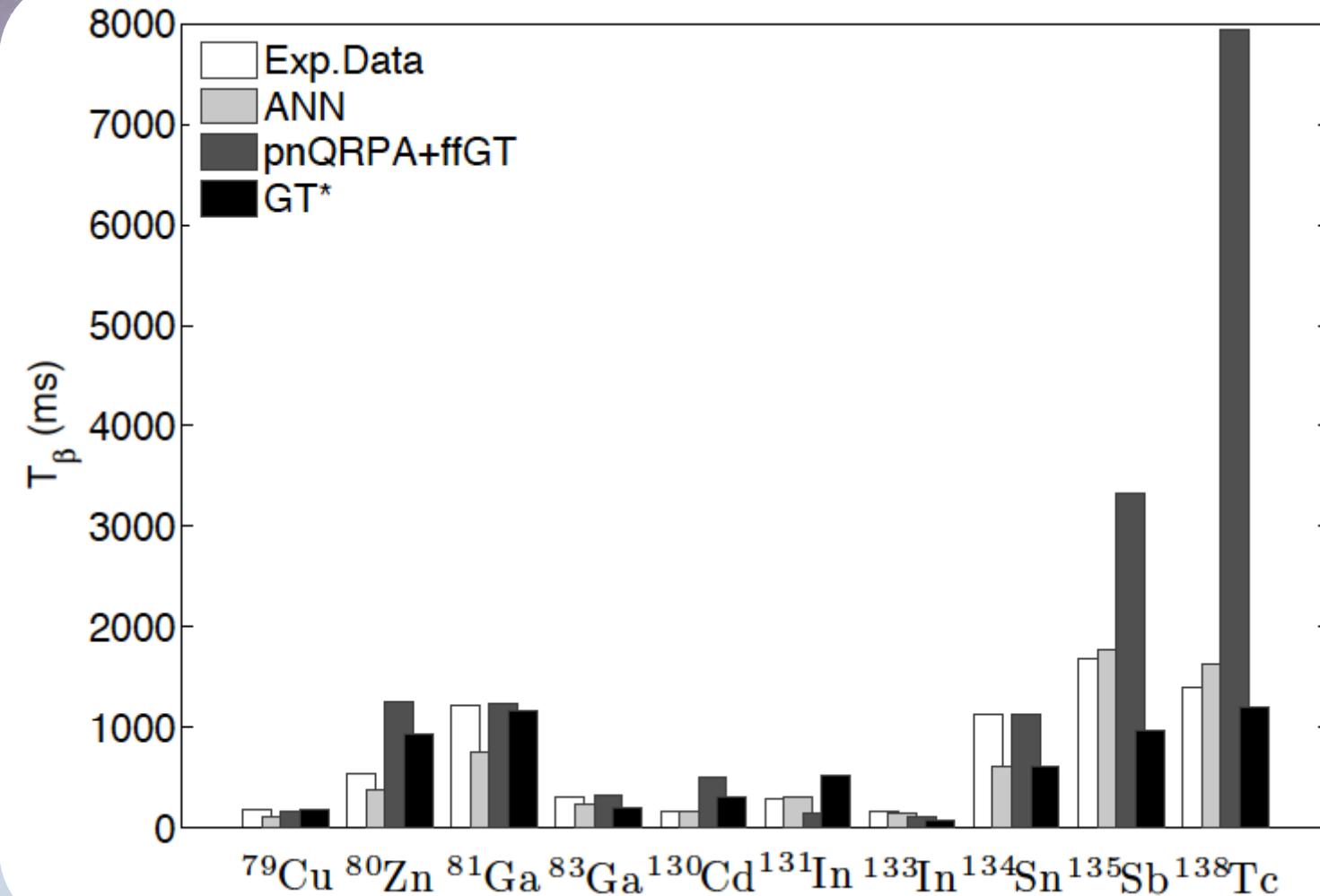


# Isotopic Chain of $^{77}\text{Ir}$



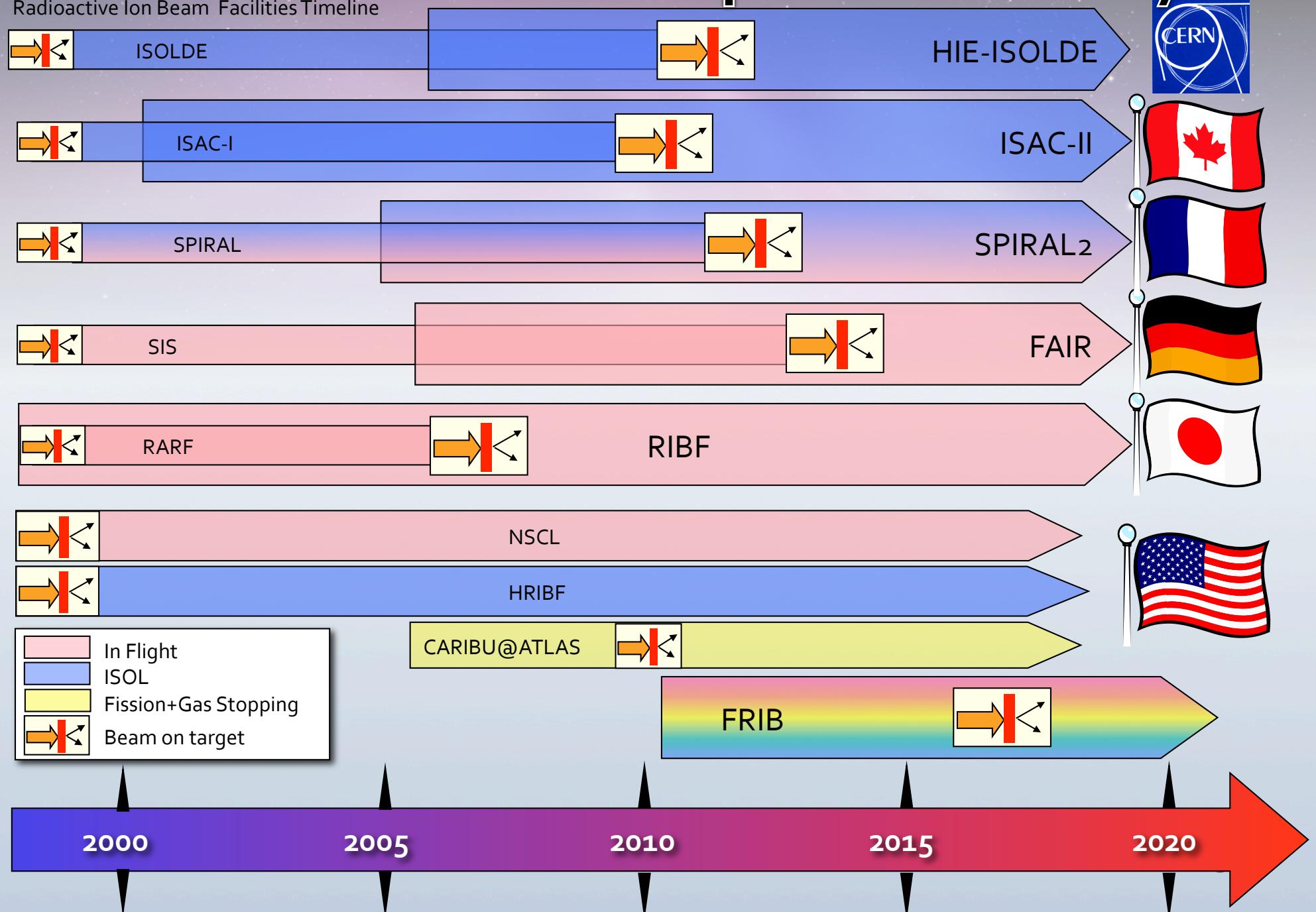
# Nuclides on or near a typical r-process path

with  $S_n$  up to 3 MeV



# New data will help enormously

Radioactive Ion Beam Facilities Timeline



# Conclusions & Prospects

- Conclusions

Theory-thin statistical global models of nuclear properties developed by learning machines should provide a valuable, robust additional tool to **complement** the nuclear systematics studies **beyond the standard nuclear landscape** planned with the present and future very neutron- rich, rare-isotope experimental facilities.

- Prospects

We plan further studies of nuclear properties relevant to r-process: masses, neutron capture cross-sections with the already developed by us ANN and SVM techniques.

We also plan further studies of nuclear properties using Artificial Intelligence's more mature learning strategies, such as **committee of machines** (CoM) - a collection of different feed-forward Artificial Neural Networks (ANNs) instead of a single ANN, with a view to refine current results.

# Questions



## Thank You

**$\beta^-$ -decay half-lives using the ANN model:  
Input for the r-process\***

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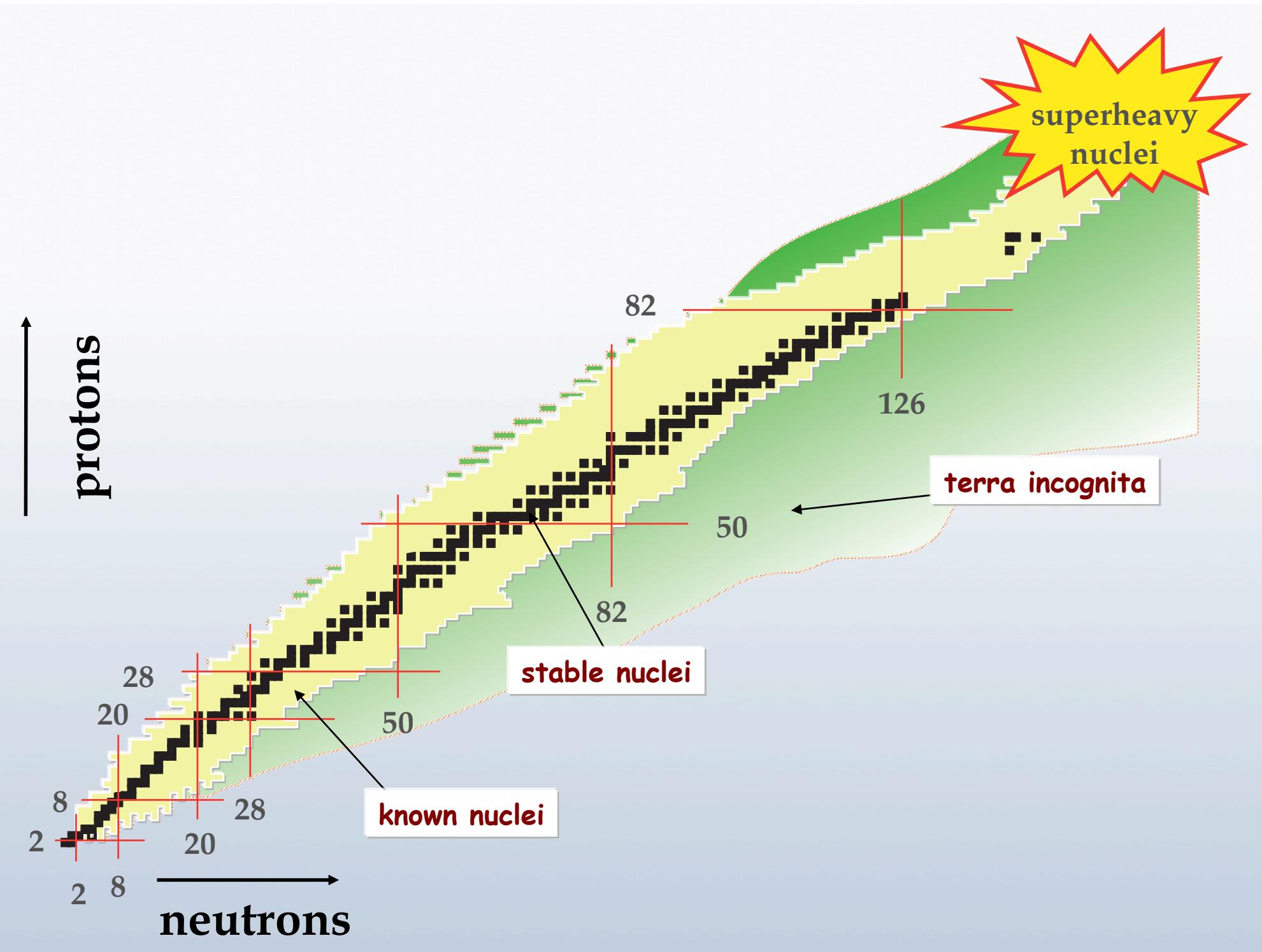
<sup>‡</sup> McDonnell Center for the Space Sciences and Department of Physics, Washington University, St. Louis, Missouri 63130, USA

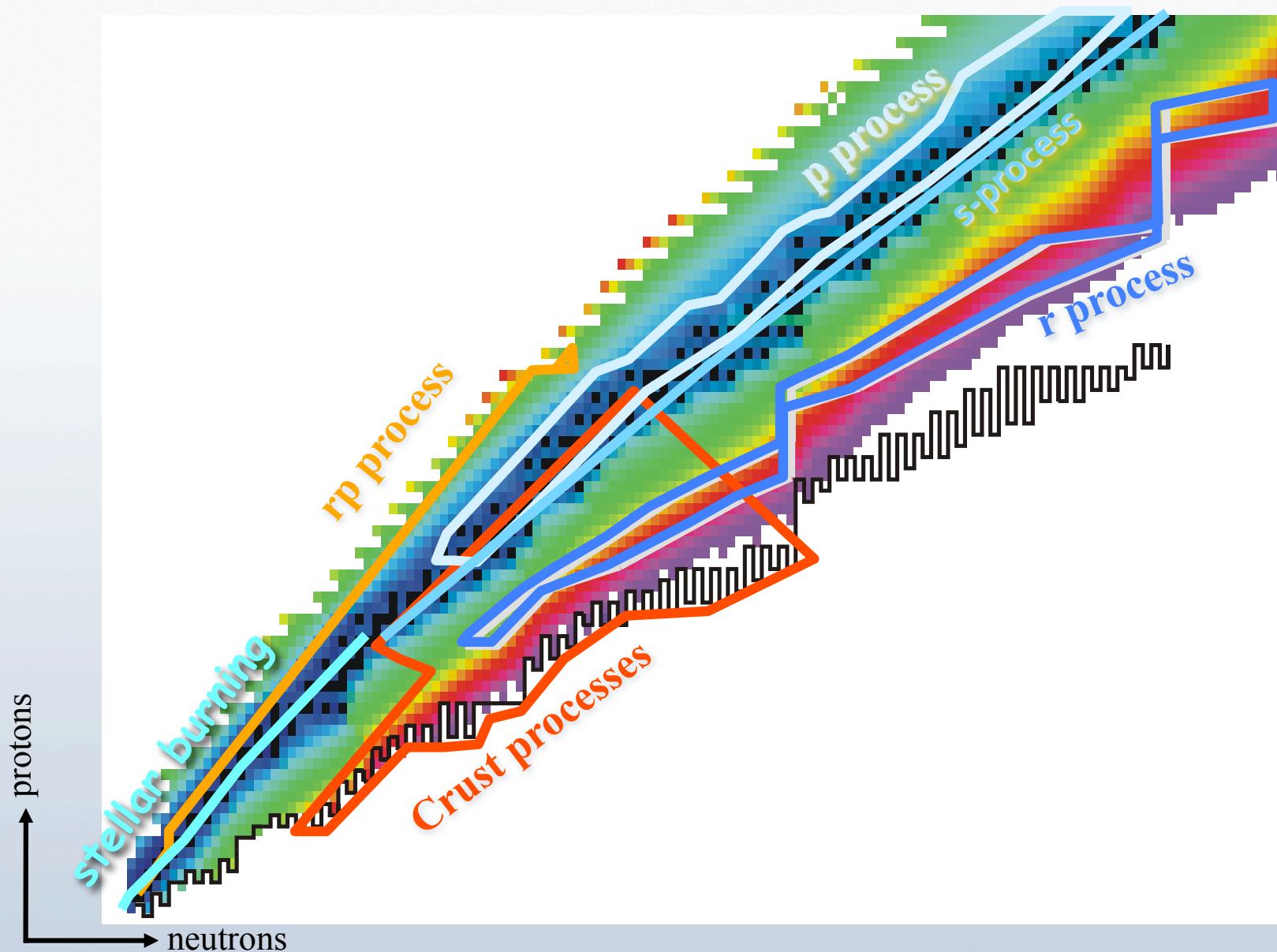
<sup>§</sup> School of Physics & Astronomy, Schuster Building, The University of Manchester, Manchester, M13 9PL, United Kingdom

URL: <http://www.pythaim.phys.uoa.gr>

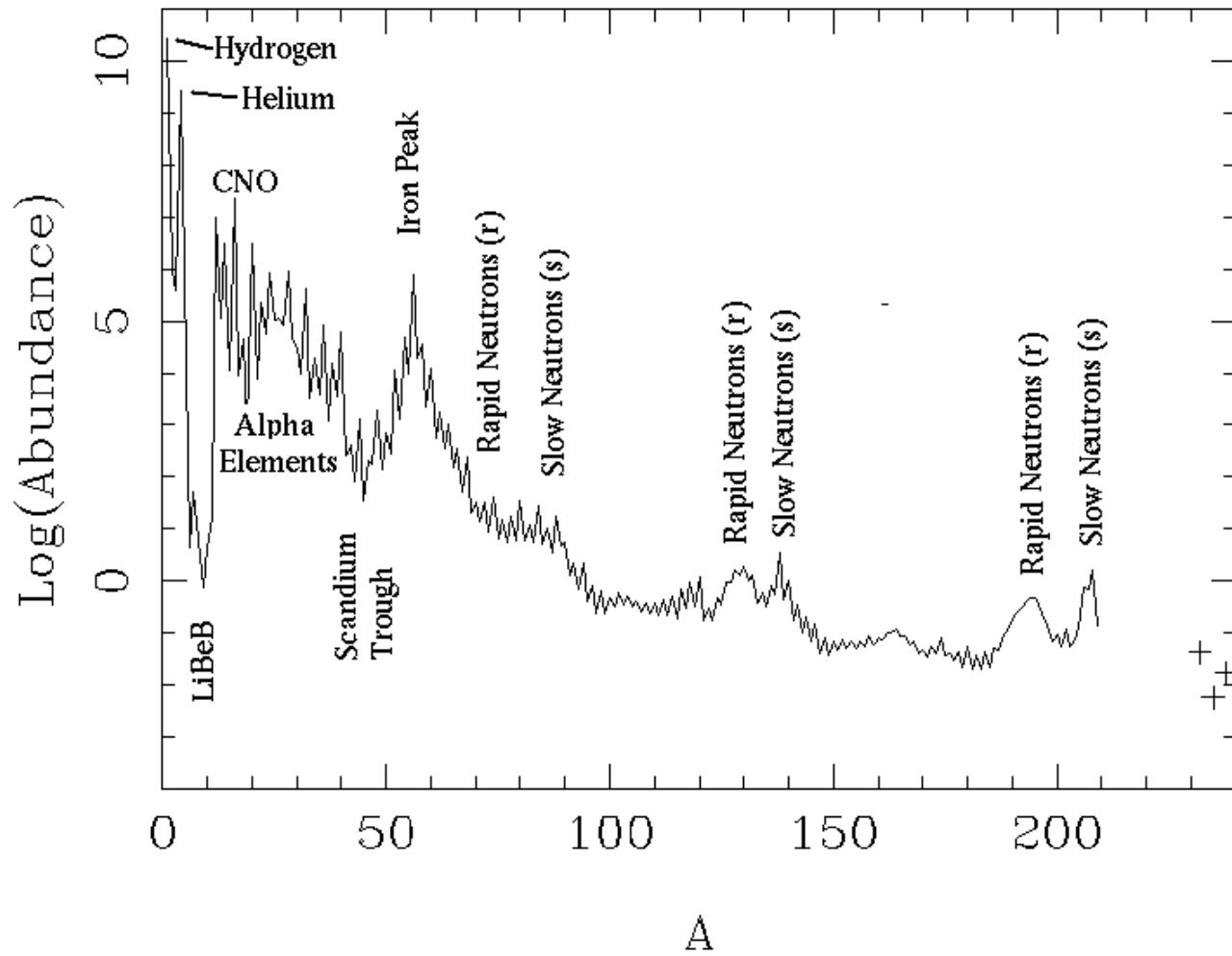
E-mail: [pythaim@phys.uoa.gr](mailto:pythaim@phys.uoa.gr)

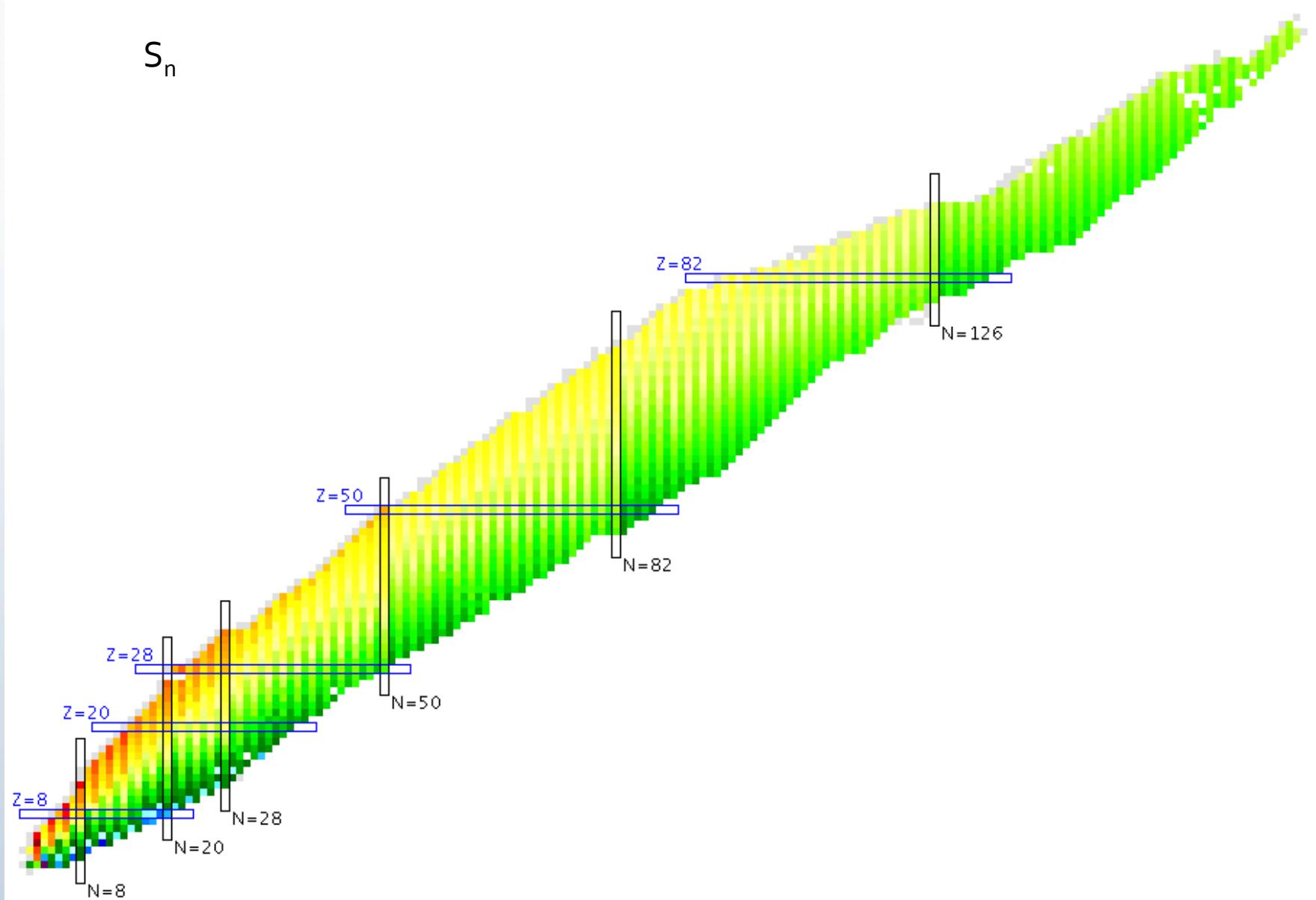
# Backup Slides

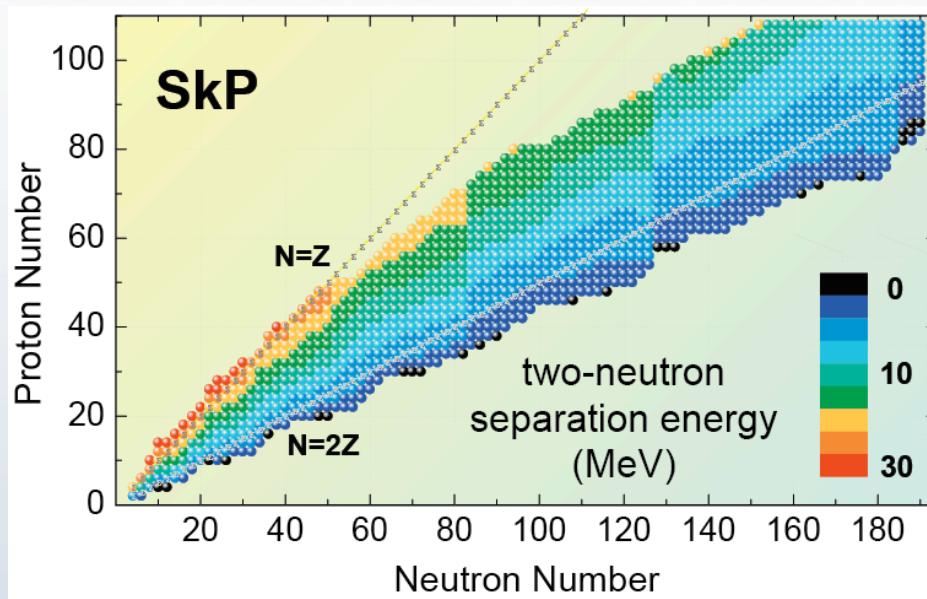


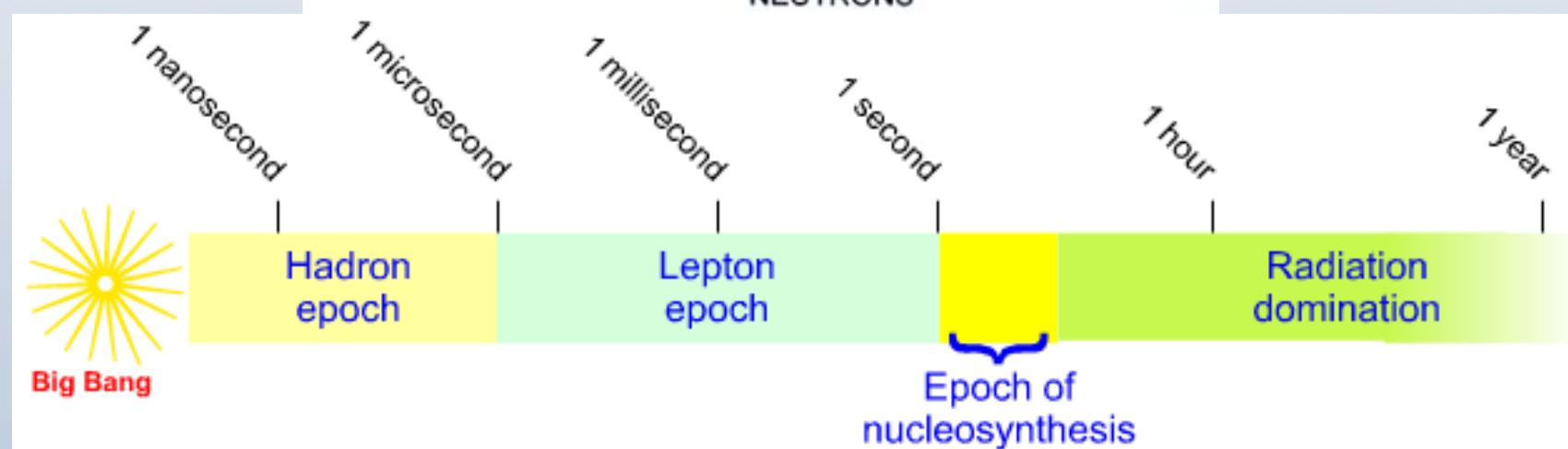
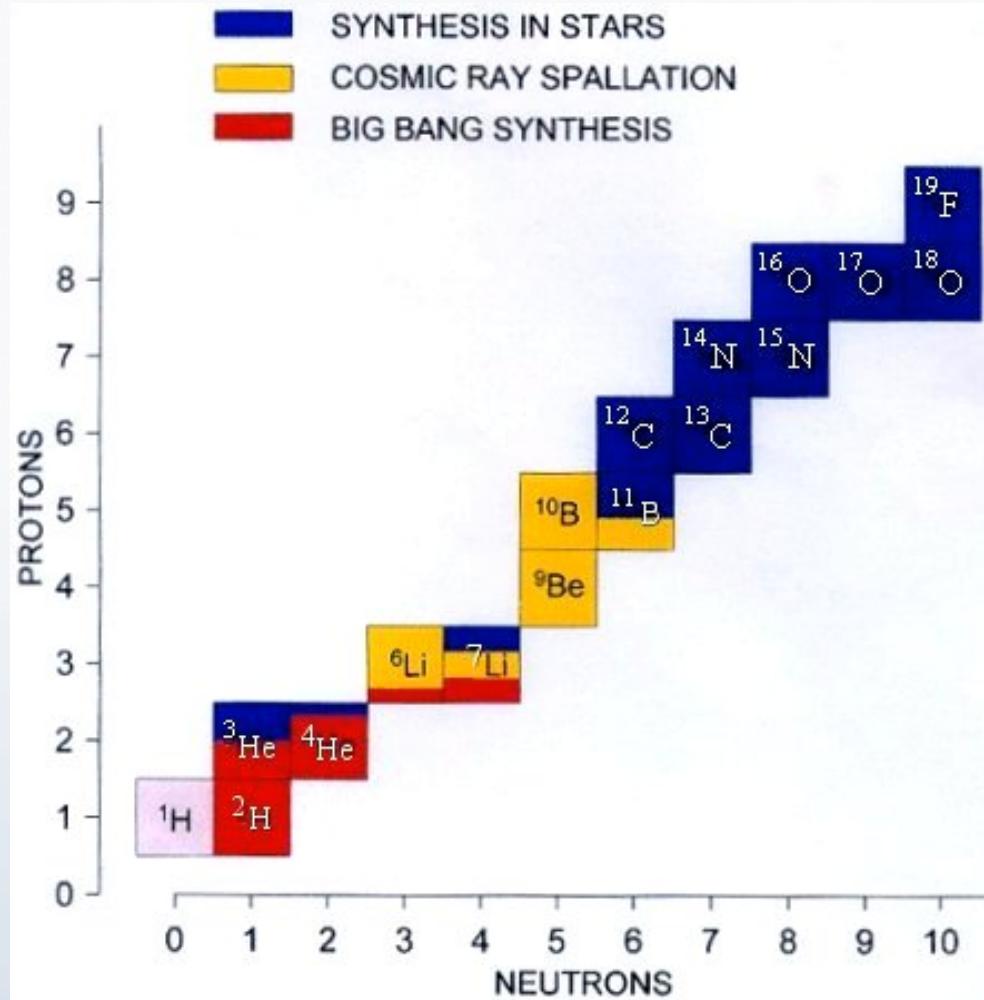


Standard Abundance Distribution (SAD) vs. A

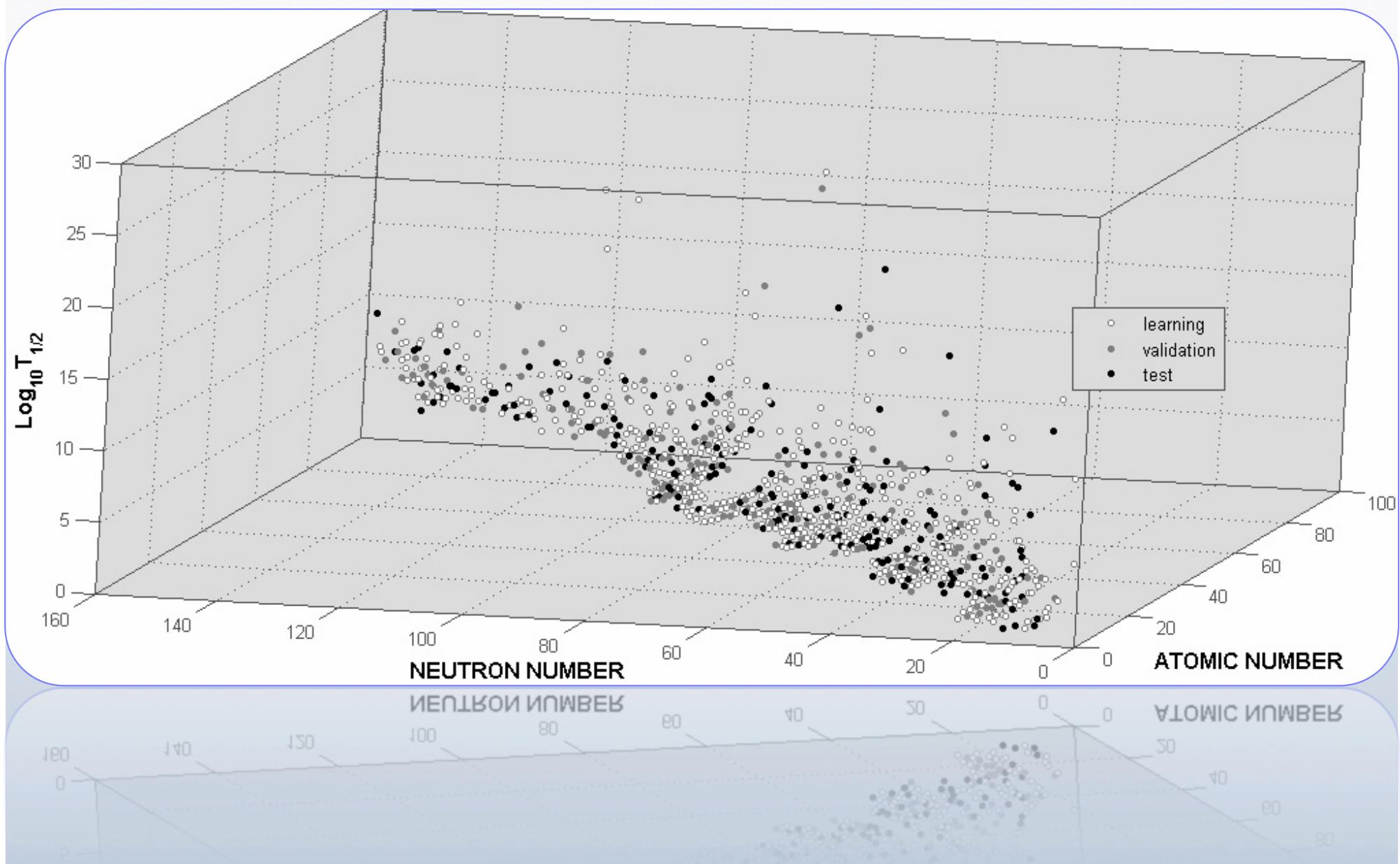




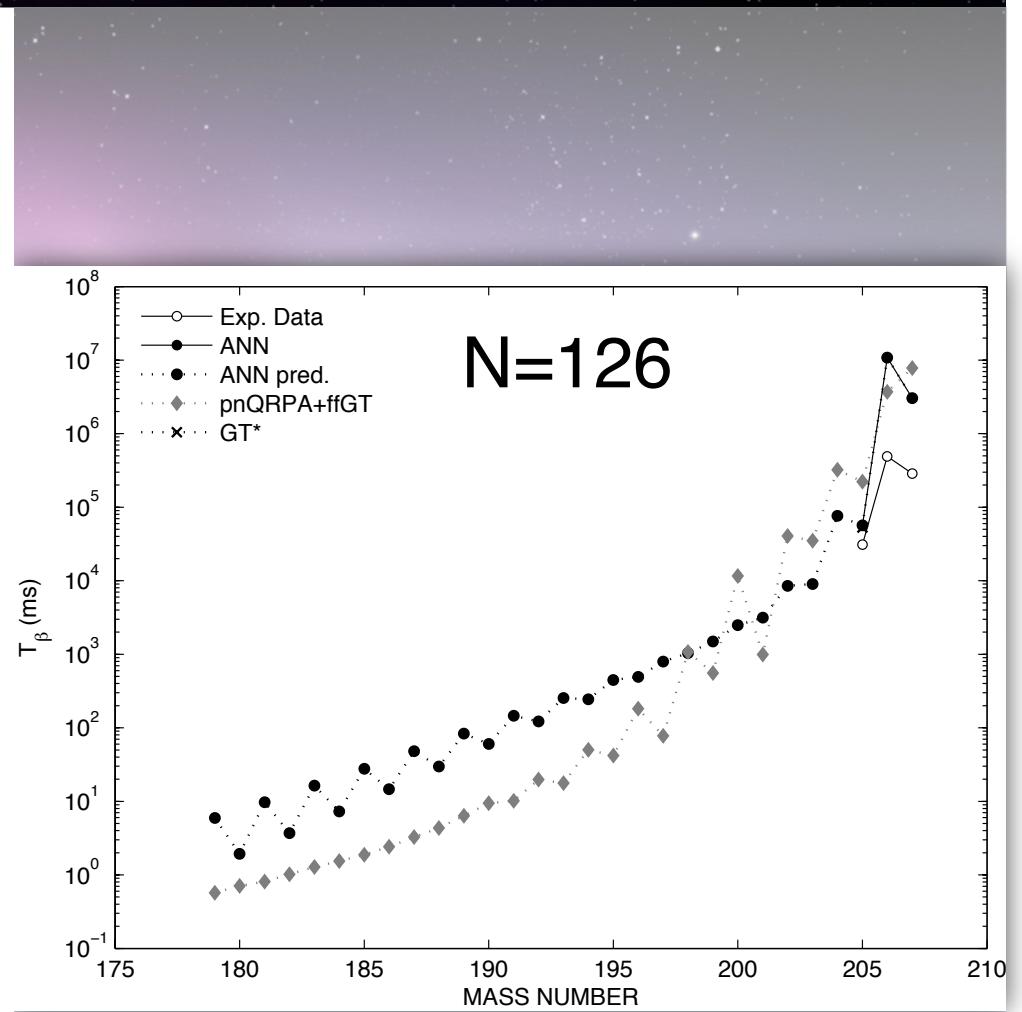
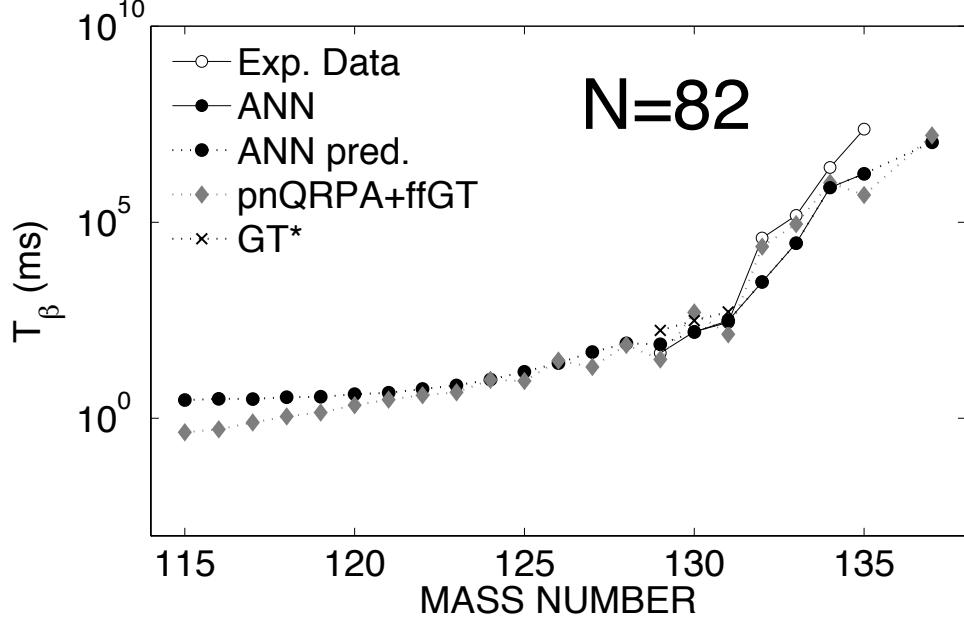
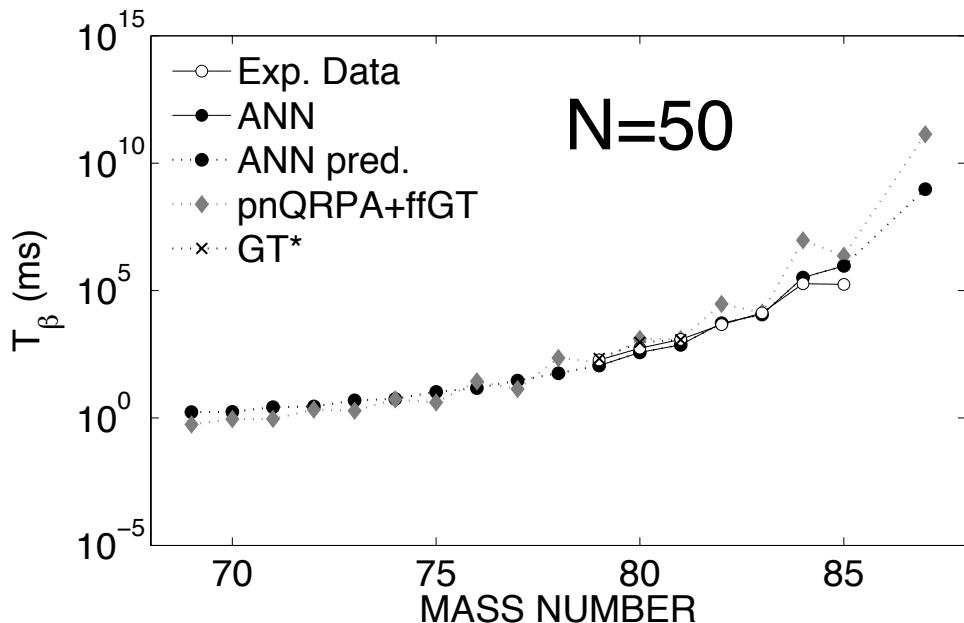




# NuBaseo3

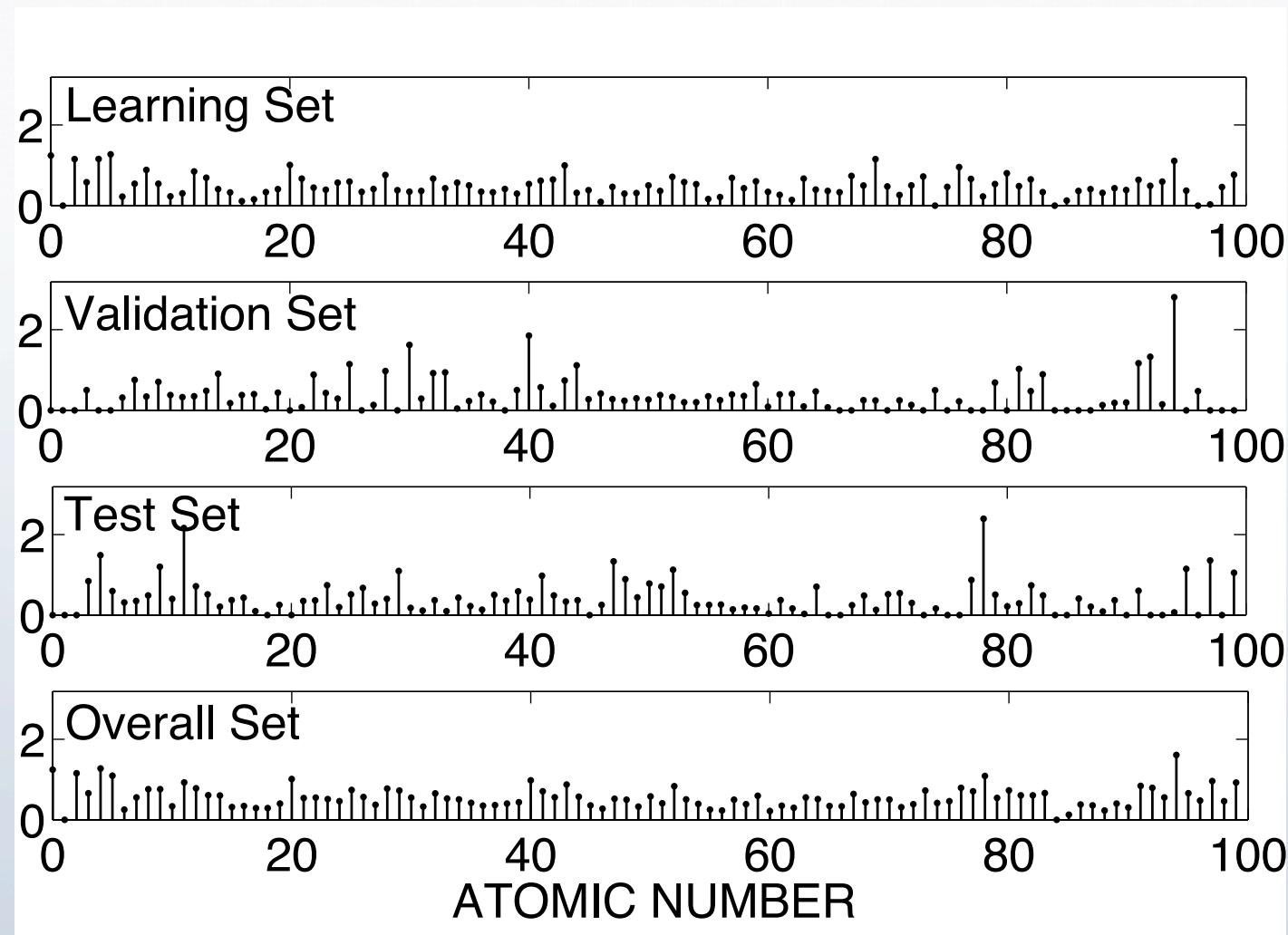


# Isotonic Chains



Nucleus	NUBASE03	ANN Model	$T_{\beta^-}(s)$	$pnQRPA+ffGT$	$pn\text{-}RQRPA$	$pn\text{-}RQRPA+ffGT$
$^{70}\text{Ni}$	6.00 (0.30)	7.58	36.72	35.28	4.89	
$^{72}\text{Ni}$	1.57 (0.05)	1.83	7.03	9.11	1.42	
$^{74}\text{Ni}$	0.680 (0.120)	0.530	1.20	3.57	0.563	
$^{76}\text{Ni}$	0.470 (0.390), 0.238 $^{+0.015}_{-0.018}$	0.164	0.433	2.13	0.279	
$^{78}\text{Ni}$	0.110 $^{+0.100}_{-0.600}$	0.057	0.224	1.71	0.150	
$^{122}\text{Cd}$	5.24 (0.03)	8.74	15.17	0.549	2.40	
$^{124}\text{Cd}$	1.25 (0.02)	1.71	15.20	0.416	1.21	
$^{126}\text{Cd}$	0.515 (0.017)	0.560	3.23	0.329	0.673	
$^{128}\text{Cd}$	0.280 (0.040)	0.265	0.695	0.297	0.429	
$^{130}\text{Cd}$	0.162 (0.007)	0.158	0.502	0.299	0.299	
$^{132}\text{Cd}$	0.097 (0.010)	0.107	0.243	0.184	0.172	
$^{198}\text{Ir}$	$8^{+2}_{-2}$	57.6	377.1	19.1	-	
$^{199}\text{Ir}$	$6^{+5}_{-4}$	73	370.6	46.7	96.6	
$^{202}\text{Ir}$	$11^{+3}_{-3}$	8.6	68.4	9.8	8.5	

RMSE



# Klapdor's Metrics\*

\* H. Homma, M. Bender et al., Phys. Rev. C 54:6 (1996) 2972

fact	$T_{1/2}$ Exp (s)	Overall Mode			Prediction Mode			NBSC+pnQRPA		
		m(%)	$\langle x \rangle_K$	$\sigma_K$	m(%)	$\langle x \rangle_K$	$\sigma_K$	m(%)	$\langle x \rangle_K$	$\sigma_K$
<10	<10^6	90.5	2.46	1.72	90.5	2.69	1.85	76.7	3.00	-
	<60	96.5	2.21	1.52	96.1	2.48	1.64	87.2	2.81	-
	<1	97.6	2.10	1.39	98	2.24	1.30	95.7	2.64	-
<5	<10^6	82.8	1.99	0.95	79.2	2.10	0.97	-	-	-
	<60	90.2	1.88	0.84	87.3	2.05	0.91	-	-	-
	<1	93.7	1.88	0.8	94	2.04	0.89	-	-	-
<2	<10^6	53.5	1.41	0.27	49.4	1.48	0.28	33.8	1.43	-
	<60	60.6	1.41	0.27	53.9	1.48	0.27	42.0	1.41	-
	<1	61.9	1.41	0.26	60	1.50	0.27	50.7	1.43	-

$\bar{x}_K$  must tends towards 1 while  $\sigma_K$  must tends towards 0.

$$\bar{x}_K = \frac{1}{N} \sum_i x_i, \quad x_i = \begin{cases} T_{\beta,\text{exp}} / T_{\beta,\text{calc}} & \text{if } T_{\beta,\text{exp}} \geq T_{\beta,\text{calc}} \\ T_{\beta,\text{calc}} / T_{\beta,\text{exp}} & \text{if } T_{\beta,\text{exp}} < T_{\beta,\text{calc}} \end{cases} \quad \sigma_K = \left[ \frac{1}{N} \sum_i (x_i - \bar{x}_K)^2 \right]^{1/2}$$